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Road Network Resilience: How to Identify Critical Links Subject to Day-to-Day Disruptions

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Abstract

Disruptive events occur on road networks on a daily basis and affect traffic flow. Resilience analysis aims to assess the consequences of such disruptions by quantifying the ability of a network to absorb and react to adverse events. In this paper, we advance a methodological approach based on resilience stress testing and a dynamic mesoscopic simulator. We aim to identify and rank the links most critical to the overall performance of the road network, taking into account the dynamic properties of road traffic and focusing on day-to-day disruptions. As a metric to quantify road network performance in the presence of such disruptions, we use the increase in overall travel cost. We then compare our approach with purely topological approaches. We discuss the advantages and drawbacks of the different analyzed metrics. We prove that link ranking varies greatly depending on the metric. Specifically, the proposed stress testing methodology can produce very accurate results by taking into account demand and congestion, but requires computationally intensive simulations, being therefore prohibitive even on medium-sized networks. Conversely, purely static topological metrics can be inaccurate if they do not take into account traffic demand and network dynamics. Our study highlights the need for joining traditional traffic-agnostic topological resilience analysis with demand-aware dynamic stress testing techniques.

Roads are vulnerable to being disrupted by adverse weather conditions, human errors, or technological breakdowns. Such day-to-day disruptions affect traffic flow and may profoundly impair the ability of the transportation network to guarantee basic mobility services as well as management of emergency situations, thus causing fundamental economic and social strains.

Resilience analysis aims to evaluate and predict the consequences of such disruptions, and has become a crucial research concern in recent years (*1*, *2*). In the field of road networks, one possible approach consists in identifying the links that most strongly affect the overall performance. Operators and planners should be aware of the consequences deriving from reduced capacities on links. They should focus their efforts on improving and maintaining such critical links, since they may cause the most severe consequences for traffic operations when disrupted. Thus, a methodological approach and a set of resilience metrics are fundamental to identify critical links.

Similar to other complex systems, a city road transportation network can be modeled as a graph $G = (N, L)$ where road intersections are represented as nodes (N) and roads as links (L). This allows for the use of graph theory and network connectivity analysis to study resilience. Graph theory can make resilience analysis very efficient in terms of computation costs, especially with the widespread adoption of big data technologies and cloud-computing. However, the phenomenon of congestion—dynamic, spatio-temporal, demand dependent—is traditionally not addressed in topological studies on road traffic resilience (*3*, *4*).

In this context, our work aims to answer the following research questions:

- How is it possible to identify the links that are most critical to the operation of the whole road network, especially with respect to day-to-day disruptions?
- Are topological metrics adequate to measure resilience for road networks?

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- Which metrics are the most effective in assessing road network resilience, taking into account their dynamic, spatio-temporal, and demand-dependent properties?

The main contributions of this paper are the following: (a) a methodology based on link-based stress testing and a dynamic mesoscopic simulator is proposed for identifying critical links and quantifying road network robustness; (b) the methodology is compared with multiple topology-based metrics to clearly identify their limitations on a simplified road network; (c) the approach is evaluated on a real-world network to identify and rank the most critical links in a realistic scenario; (d) guidelines are provided to decide when one approach can be preferred to another.

The paper is outlined as follows. The next section deals with the related literature. In the third section, we review different resilience metrics and describe our methodological approach, by presenting the stress testing approach and our dynamic mesoscopic simulator. The fourth section describes our two case studies. The fifth section reports on the evaluation of our approach in the considered case studies. A final discussion is then presented, while some directions for further work are proposed in the final section.

Literature Review

In this section, we firstly introduce the very general concept of resilience based on previous works, by adapting it to the context of road network analysis. We then present relevant road network resilience approaches for identifying critical links.

Resilience Definition

A wide diversity of definitions has been introduced in the literature to characterize resilience. Woods identified four major concepts (*5*): (i) resilience as rebound from trauma and return to equilibrium; (ii) resilience as a synonym for robustness; (iii) resilience as the opposite of brittleness, that is, graceful extensibility when surprise challenges boundaries; and (iv) resilience as network architectures that can sustain the ability to adapt to future surprises as conditions evolve.

Bruneau et al. (*6*) define resilience as the ability to: (i) mitigate hazards (*robustness* or *pre-perturbation resilience*); (ii) contain the effects of disasters during their occurrence (*reactivity*); and (iii) identify disruptions rapidly as they occur and mobilize resources to recover an acceptable traffic flow quickly (*recovery* or *post-perturbation resilience*).

Sullivan et al. define road network robustness as the degree to which the network can function in the presence of capacity disruptions on links. A robust road network can face disruptions on links with only slight increases in overall network-wide travel costs. Conversely, a non-robust road network is subject to substantial increases in costs (*7*).

In the rest of the paper, we tackle the problem of resilience assessment according to the perspective provided by Woods's second concept, that is, *robustness* (*5*), and the definition proposed by Sullivan et al (*7*). The paper proposes an approach based on stress testing for assessing the impact of day-to-day disruptions on network links by measuring overall travel costs, aiming to quantify the ability of the whole road network to absorb such an impact. We remark, as an additional contribution of this paper, that our approach also allows for assessment of resilience under the occurrence of unexpected and possibly very rare events, which corresponds to Woods's third concept that is, graceful extensibility when surprise challenges boundaries (*5*). Our stress testing methodology is also applied for evaluating the impact of sudden variations in travel demand on the overall travel costs.

Approaches to Assessment of Resilience

The majority of the approaches aimed at quantifying resilience are based on topology models and static network connectivity analysis (*3*, *4*, *8*). Among the plethora of metrics proposed to perform connectivity analysis, betweenness centrality (BC) is traditionally the best choice for traffic network analysis purposes, as it expresses the frequency with which a point falls between pairs of other points on the shortest paths connecting them (*9*). BC has therefore been widely adopted to assess road resilience, by identifying topologically vulnerable links and intersections (*10*, *11*). Very recently, approaches based on machine learning and big data solutions (*12*) have been proposed to significantly reduce computation time of BC on several kinds of very large complex networks (*13*).

Even though a large majority of studies focus on static topological features of the network to identify its vulnerabilities, some authors have tried to join network topology features to more dynamic traffic-related information. Augmented definitions of BC take into account time-varying origin–destination (OD) travel demand (*14–17*), as well as travel times (*16*, *18*, *19*) for shortest paths computation. Such augmented metrics have been adopted for traffic flow analysis and prediction (*14*, *15*, *18*, *19*) and traffic assignment (*16*) as well as network performance monitoring in the presence of extreme events (*17*). To the best of our knowledge, none of the previous approaches has been evaluated with respect to day-to-day disruptions (i.e., reduced link capacity), which represents one of the core aspects of our paper. Also, their applicability to resilience assessment is still neither widespread nor fully understood, these metrics often being inaccurate in highly dynamic environments (*20*) and prohibitive to compute for large-scale networks (*12*).

Another contribution of our paper is a methodology based on stress testing, that is, pushing a system beyond its normal operational capacity and observing how it responds to the applied stress. Stress tests have been widely used in banking (*21*, *22*), medical (*23*, *24*), and hydro-geology domains (*25*). Also referred to as network-disruption analysis, this approach has been occasionally leveraged in the field of transportation to identify critical links in a road network, but it is still at a very early stage (*17*, *26*, *27*). Sullivan et al. (*7*) set different link-based capacity-disruption values for identifying and ranking the most critical links and quantifying road network resilience. Jenelius and Mattsson (*28*) developed the notion of the importance of a link, which is a function of the increase in travel time when the link is disrupted. Their method is demand-aware as they weight travel time by demand. Stress testing allows intra-network comparison; links are ranked based on their contribution to the overall network resilience (*29*, *30*). Other authors have focused on identifying only the most critical nodes to be improved (*30*). Some studies use probability-based models to calculate the likelihood that a network continues functioning after a given stress (*31*, *32*). To the best of our knowledge, the large majority of research works based on stress testing for assessing the resilience of road networks mainly deal with disasters and extreme events (*17*, *33*, *34*), instead of day-to-day disruptions as targeted in this paper.

Assessment of the Literature

The definition and quantification of resilience varies greatly depending on the context and application domain. No universal and totally agreed definition or metric of resilience exists. Despite the plethora of work on resilience in various domains, relatively small numbers of studies have targeted resilience of road networks explicitly, and very few applications on real-world roads have been proposed. Our paper deals instead with a real network in France, in the Paris agglomeration area.

The concept of resilience can be divided into two parts: pre-perturbation resilience (robustness), and post-perturbation resilience. In this study, focused on traffic modeling and analysis, we evaluate pre-perturbation resilience only. We do not account for post-perturbation socio-technical actions. As the aim of this paper is to rank and identify the most critical links in a given network with respect to day-to-day disruptions, we focus on intra-network comparisons.

In the literature, two general approaches are commonly used to quantify resilience (*26*). The first is purely topological and usually demand-insensitive. The second takes into account traffic network performance and includes demand variability via simulation.

It is worth noting the lack of studies combining the stress testing approach with the topological one. We analyze and compare the two different approaches to assess the resilience of road networks, and to determine their advantages and drawbacks in assessing road network resilience. For the first approach, that is, the topological one, we consider and evaluate several metrics from graph theory. As for the second approach, by means of a simulator, we inject perturbations in the network and quantify to what degree it can adjust to them.

Methodology

In the following, we report on the topological metrics considered in our analysis as well as the stress testing technique proposed to perform demand-aware, dynamic disruption analysis.

Graph Theory Metrics

The topological metrics used in this study are based on BC, originally proposed by Freeman (*8*). BC measures the importance of the generic link l of a graph by considering the number of shortest paths that traverse it, and is defined as follows:

$$BC(l) = \sum_{i \neq l \neq j} \frac{\sigma_{ij}(l)}{\sigma_{ij}}, \tag{1}$$

where:

- $\sigma_{ij}(l)$ is the number of shortest paths from node i to node j that traverse link l;
- σ_{ij} is the total number of shortest paths from node i to node j.

In shortest path computation, links can be unweighted or weighted (e.g., in terms of the associated estimated travel time). In this study, we test and compare both cases. We also consider multiple variants of the BC, reported in the following, to model different aspects of a road traffic network.

BC for Entry and Exit Nodes Only. We propose to use an alternative definition of BC consisting in calculating the shortest paths from entry to exit nodes only. (The computation of BC on a subset of nodes [entries and exits] is based on the function *edge_betweenness_centrality_subset* from the *NetworkX* Python library.) This definition introduces two advantages: computation time is reduced, and the definition seems more realistic from a demand-aware perspective, since individuals tend to start and finish their trips over a subset of intersections. This corresponds to the OD representation of the traffic demand. The formula is the same as Equation 1 with some exceptions:

- i is selected from the entry-nodes subset, i.e., intersections used by vehicles to enter the network;
- j is selected from the exit-nodes subset, i.e., intersections used by vehicles to leave the network.

In conclusion, we consider four different formulations of the BC:

- Unweighted BC (BC)
- Travel-time weighted BC (TTWBC)
- Unweighted BC on entry/exit nodes only (BC entries–exits)
- Travel-time weighted BC from entry to exit nodes only (TTWBC entries–exits)

Spatio-temporal traffic properties and phenomena, like demand, congestion and dynamic re-routing, are typically not addressed in graph-based models. Thus, graph-based metrics are usually incapable of capturing these aspects in turn.

Demand-Sensitive Metric

Jenelius et al. (*2*) introduced the demand-aware metric of importance (I) to characterize transportation vulnerability. This metric allows measurement of network performance loss by using travel costs weighted by the traffic demand. Such metric is adequate for our methodological approach as it includes demand and the dynamic phenomenon of congestion (i.e., travel costs increase when traffic is congested). This metric uses a generic notion of travel cost, which can be specified depending on the study context and aim. In this paper, we define travel cost as travel time divided by travel distance (in seconds/kilometer). The importance of a link l is the following:

$$I(l) = \frac{\sum_i \sum_{j \neq i} x_{ij}(c_{ij}^{\delta}(l) - c_{ij}^{0})}{\sum_i \sum_{j \neq i} x_{ij}}, \tag{2}$$

where:

- x_{ij} is the demand from origin node i to destination node j (number of vehicles);
- $c_{ij}^{\delta}(l)$ is the mean travel cost from origin node i to destination node j when link l is disrupted at level δ;
- c_{ij}^{0} is the mean travel cost from nodes i to j in the base case, i.e., without disruption.

Traffic Model and Algorithms

To model traffic dynamics we use a dynamic mesoscopic simulator based on the Lighthill-Whitham-Richards model (*35*, *36*) and implemented in Matlab by our research group (*37*, *38*). The Lighthill-Whitham-Richards model is formulated in Lagrangian-space coordinates and uses both Lagrangian and Eulerian observations. It represents individual vehicles but only records their transit times at network nodes. A dynamic traffic assignment procedure distributes vehicles along all the possible alternative paths in the network, according to the traffic conditions at the moment the vehicle is generated. More precisely, travel times on all paths are calculated based on traffic flow, and the vehicle chooses the path that requires the smallest travel time. The following parameters have to be specified before running simulations: simulation duration, OD demands, and link capacities. This simulator is adequate for our stress testing approach: as opposed to static topological indicators, it includes traffic dynamic properties such as demand, congestion, traffic-based route assignment, dynamic shortest path computation, and queues. Moreover, travel costs are calculated for each vehicle and can be extracted easily to compute network performance metrics, such as importance from Equation 2.

Stress Testing

The aim of our stress testing methodology is to identify the most critical links in the road network and to assess their resilience by considering the dynamic, spatio-temporal, and demand-dependent properties of network traffic.

Stress testing can be leveraged to quantify the adverse impacts associated with a reduction of capacity on specific links. Disruptive road events such as flooding, obstacles on the road, and traffic accidents are likely to reduce the capacity of a given link and negatively affect network performance. Measuring network performance loss when reducing the capacity of a given link provides the criticality of this link to the operation of the whole network. Therefore, stress testing is an adequate methodological approach to identify and rank the most critical links. It captures the relative importance of the disrupted link to the other links and assess the overall resilience of the whole road network from an intra-network comparison.

Our methodology for road network stress testing is composed of the following steps:

1. *Simulating disruptive road events*:

We propose two strategies to perform this step. In the first one, we simulate day-to-day road disruption as link capacity drops. The *capacity-disruption level* is defined as the reduction in link capacity, expressed as a fraction of the original one. In many studies, the capacity-disruption level is total, that is, 100% of the original value, which means that the capacity of the link is reduced to 0 vehicles per hour (*10*). The link is then completely removed from the road network. However, a 100% capacity-disruption level does not accurately reflect the actual link capacity resulting from frequent day-to-day disruptions or minor events (e.g., number of lane reductions, adverse weather, etc.) that can affect the network. That is why we gradually reduce the capacity to analyze the evolution of the performance depending on the capacity-disruption level. We consider five possible capacity-disruption levels, denoted as δ, on each examined link, that is, $\delta \in \{0\%, 20\%, 40\%, 60\%, 80\%\}$. Therefore, we can consider the following equation to compute the maximum capacity of the generic link l in the presence of a capacity-disruption level δ:

$$\delta = 100 \cdot (1 - \frac{q^{\delta}_{max}(l)}{q^{0}_{max}(l)}), \tag{3}$$

where:

- δ is the capacity-disruption level applied to link l (percentage) with $\delta \in \{0, 20, 40, 60, 80\}$;
- $q^{\delta}_{max}(l)$ is the capacity of link l when it is disrupted at level δ (in vehicles/hour);
- $q^{0}_{max}(l)$ is the capacity of link l in the base case (in vehicles/hour);

As a second strategy to simulate disruptive road events, we consider increases in the traffic demand on specific entry/exit nodes of the network. By this approach, it is possible to simulate exceptional situations like city evacuations following extreme events (e.g., flooding, attacks, etc.) which typically put significant strain on the road infrastructure and result in total congestion. This strategy consists in changing the OD matrix, that is, increasing the traffic flow from given entries, and comparing the stress testing results with another demand level.

Based on the selected strategy for disruptive road events, we set the parameters of our mesoscopic simulator (e.g., link capacity, traffic demand) and we simulate the network in the specific setting. For both strategies, travel costs are collected for all vehicles in order to compute the performance metrics described in the following point.

2. *Computing overall performance loss*:

This step is about quantifying the consequences of the simulated disruptive event on the operation of the whole network. To this purpose, we use the notion of road network performance, measured via the importance metric of Equation 2. Specifically, we consider travel time increase divided by travel distance as a measure of cost. Below, we provide the formulation of the overall performance loss (*PL*), based on the notion of importance, when a link l is disrupted:

$$PL(l, \delta) = \sum_{v=1}^{n} \frac{c^{\delta}_{v}(l) - c^{0}_{v}}{n} \tag{4}$$

where:

- $PL(l, \delta)$ is the overall performance loss when link l is disrupted at level δ (seconds/kilometer);
- $c^{\delta}_{v}(l)$ is the travel cost of vehicle v when link l is disrupted at level δ (seconds/kilometer);
- c^{0}_{v} is the travel cost of vehicle v in the base case (seconds/kilometer);
- n is the number of vehicles in the network.

3. *Analyzing the results*:

For each link we know the performance loss corresponding to the considered capacity-disruption level (i.e., 0%, 20%, 40%, 60%, and 80% of the original link capacity). As an example, $PL(2, 40\%)$ represents the overall performance loss on link 2 when it is disrupted at 40% of its initial capacity. In the evaluation section, we present and discuss the overall performance loss depending on the capacity-disruption level of the link according to two different strategies.

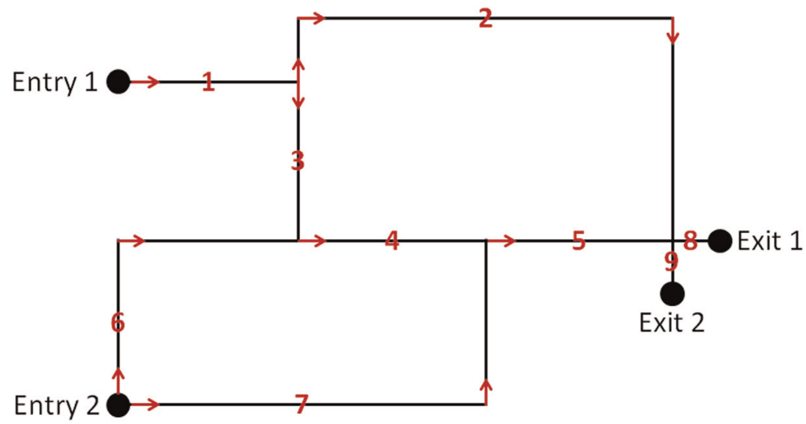
Traffic Robustness Index

The stress testing methodology reported in the previous section allows us to compute the overall performance loss for each link of the network with respect to five different capacity-disruption levels. To compare and identify critical links, we need a unique value of criticality associated to each link. To this purpose, a global metric is required to aggregate the performance loss values in the five different capacity-disruption levels. We propose the stress test criticality (STC) metric, defined as follows, for the generic link l:

$$STC(l) = \int_{\delta} PL(l, \delta), \tag{5}$$

where:

- $STC(l)$ is STC when link l is stress tested (seconds/kilometer)



(a) Simple virtual road network

Origin \ Destination	Exit 1	Exit 2
Entry 1	375	375
Entry 2	250	1000

(b) OD demand (vehicles/hour) for the simple virtual network (strategy A)

Origin \ Destination	Exit 1	Exit 2
Entry 1	500	500
Entry 2	1200	300

(c) OD demand (vehicles/hour) for the simple virtual network in (strategy B)

Figure 1. Simple virtual network. Network structure (*a*) and OD travel demand (*b*, *c*) for the two considered strategies.

- δ is the capacity-disruption level of link l (percentage)
- $PL(l, \delta)$ is the overall performance loss (seconds/kilometer)

We use the trapezoid rule to approximate the integral in Equation 5 from the (five) overall performance values computed on link l.

Case Studies

The methodology and metrics described in the previous sections have been evaluated in two different case studies. The first one is related to a simple virtual network, used as a basic testbed for our approach. The second one is a real road network in France, which we use to confirm the validity of our results in a realistic scenario and to support the discussion on both simulation-based stress testing and topological metrics.

A Simple Virtual Road Network

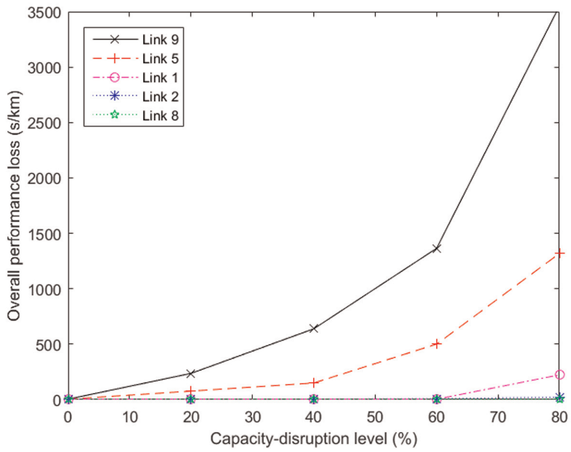
This network is composed of eight nodes (4 road intersections, 2 entries, and 2 exits) and nine links. The duration of each simulation is fixed to 10 minutes. Figure 1*a* depicts the network structure with numbered links and flow directions. The network has been tested with two different demand levels reported as OD matrices in Figure 1*b* and *c*.

DIRIF: A Real-World Road Network

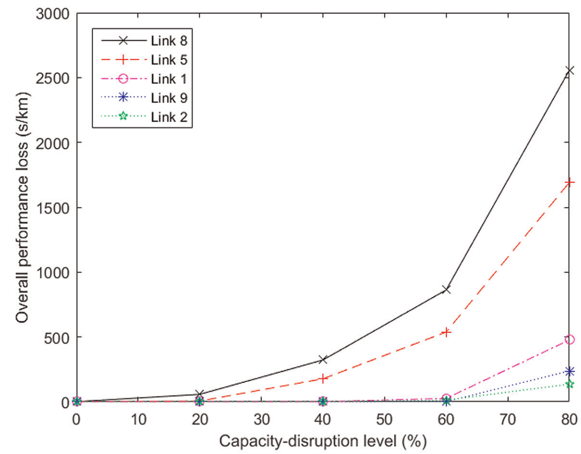
The DIRIF network is situated in the south of Paris, France, and includes 868 links and 827 nodes (657 intersections, 86 entries, and 84 exits). Its roads are mostly highways. Each simulation duration is fixed to 15 minutes. Since the network is much bigger than the virtual one described above, and traffic demand is extremely low on some links, we specify a higher simulation duration to ensure that enough vehicles travel through the whole network and to collect a significant number of travel cost observations. Simulation is performed with real demand data from 9:00 a.m. to 9:15 a.m., that is, the morning peak-time, in order to increase the probability of observing some performance loss in our stress tests. The network is graphically presented in Figure 3*a* and *b*.

Evaluation

In this section, we present the results of our stress testing methodology and discuss the link ranking derived from the different selected metrics. We show that the ranking



(a) Overall Performance Loss, strategy A (STC A)



(b) Overall Performance Loss, strategy B (STC B)

Link	STC A	STC B	BC	BC entries-exits	TTWBC	TTWBC entries-exits
9	1^{st}	4^{th}	2^{nd}	1^{st}	4^{th}	3^{rd}
5	2^{nd}	2^{nd}	1^{st}	1^{st}	1^{st}	1^{st}
1	3^{rd}	3^{rd}	2^{nd}	1^{st}	4^{th}	3^{rd}
2	4^{th}	5^{th}	2^{nd}	1^{st}	7^{th}	5^{th}
8	5^{th}	1^{st}	2^{nd}	1^{st}	4^{th}	3^{rd}
6	5^{th}	6^{th}	4^{th}	2^{th}	5^{th}	4^{th}
4	5^{th}	7^{th}	2^{nd}	2^{th}	2^{nd}	2^{nd}
7	5^{th}	8^{th}	3^{th}	1^{st}	6^{th}	4^{th}
3	5^{th}	8^{th}	3^{th}	2^{th}	3^{rd}	3^{rd}

(c) Simple network link rankings generated by the different metrics of criticality

Figure 2. Performance evaluation on the simple virtual network.

of critical links on the same network can vary significantly when different indicators are used, thus proving that simple modifications of one centrality indicator can have a relevant impact on the capacity of the metric to capture different facets of resilience. Moreover, we discuss the advantages and drawbacks of each different approach in assessing road network resilience, and provide guidelines that can be helpful towards the definition of a new enhanced centrality metric.

Application to a Simple Virtual Network

In the scenario of the simple network described in the previous section, we measured STC and all of the proposed topological metrics on all the links. To perform stress testing, we used both strategies described above, that is, link capacity drop (referred to as A in the following) and traffic demand increase (referred to as B). The measures of STC that result from the two strategies above are distinguished as STC A and STC B, respectively. STC A and B are calculated with the same formula, but different parameters are set before stress testing.

First we discuss the results of strategy A. In our simulations, we applied sequentially five capacity-disruption levels (i.e., 0%, 20%, 40%, 60%, 80%) to each link. We then measured the network-wide performance loss (i.e., Equation 4) consequent to the disruption applied to the link. The overall performance loss from our stress tests is reported on the y-axis of Figure 2a, while the corresponding capacity-disruption levels (δ) are reported on the x-axis. Results for different links are depicted with different colors and markers in the figure, using a linear interpolation. For readability, the figure only reports the five most critical links (i.e., those with the highest overall performance loss).

Intuitively, a link capacity drop translates into an increase of network-wide travel cost.

According to the definition provided in Equation 5, the STC of a given link corresponds to the area below its curve. Thus, in strategy A, *link 9* is the most critical to the operation of the whole road network, followed by *links 5, 1, 2*, and 8. By using link ranking from STC as a baseline, we compare in the following the other link rankings as derived from the different selected topological metrics. Figure 2*c* reports on such link rankings for both STC and the whole set of topological metrics.

As a preliminary consideration, it can be observed that the rankings of critical links on the same network may dramatically change depending on the metric, due to the different properties of the network captured by each of them. As an example, *link 5* is on top of all the topological rankings whereas, with STC, it is ranked second, below *link 9*. The top-rank of *link 5* by all the topological metrics can be explained by the large number of shortest paths traversing this link: e.g., paths (*4*, *5*), (*7*, *5*), (*7*, *5*, *8*), (*7*, *5*, *9*), (*6*, *4*, *5*), (*7*, *5*, *8*) are all shortest paths.

The different ranking issued by STC A can be easily explained. If *link 5* is disrupted, alternative paths through *link 2* exist for all individuals heading to *exit 1* or *exit 2*. Conversely, when *link 9* is disrupted, no alternative path exists for users willing to travel to *exit 2* from both *entry 1* and *entry 2*, thus resulting in serious congestion and increased travel time for all individuals heading to *exit 2*. Additionally, traffic demand for *exit 2* is very high (see OD matrix in Figure 1*b*). This explains why *link 5* is more critical than *link 9* in terms of topology, but less critical than *link 9* when considering demand data, as made possible by our stress testing methodology (based on dynamic simulations) and captured by the related criticality metric. This simple test clarifies how traditional demand-agnostic approaches may fail in properly ranking edge criticality.

Our simple test also shows that alternative paths may become shortest paths of the network as links are disrupted by adverse event, thus attracting traffic flow previously directed through the disrupted links. This represents another fundamental aspect that is impossible to capture with a static graph-based approach. However, this does not necessarily mean that topological metrics are not good resilience indicators, but rather that road graph modeling should include a dynamic component (e.g., edge weights), and that BC metrics should be rapidly re-computed after relevant network disruptions.

Another striking difference worth analyzing concerns *link 2*: considered as one of the most critical ones according to the BC metric from entries to exits, it is the least critical one for the TTWBC and the TTWBC from entries to exits. The peculiarity of *link 2* is its length; it is the longest one, thus demanding more time to be travelled than the other links. Metrics like BC and BC from entries to exits are not weighted, that is, all links are valued equally, and consequently they are unable to grasp this important aspect. In contrast, links with high travel times are not considered critical by the analyzed weighted approaches, because they are not often part of shortest paths. The same consideration applies to *link 7*, which is the second-longest link of the network.

Finally, it is worth noting that BC values are often clustered. In particular, the BC from entries to exits has many equal values and only two link ranks. Traditional topological metrics appear to have very limited capability to discriminate link criticality at a fine level. In this case, STC does not differentiate all links either, but this is due to capacity-disruption levels. For *links 8, 6, 4, 7*, and *3* which are all ranked in fifth place with the same value, both capacity-disruption and demand levels are not high enough to observe a significant performance loss compared with the base case. For example *link 8* capacity disruption does not affect the overall network performance (see Figure 2*a*). The overall travel cost remains at its base case value. That is why some links have the same criticality value. STC differentiation between links then depends on capacity-disruption and demand levels.

To investigate this aspect further, we use our second stress testing strategy B, that is, with different traffic demand as reported in Figure 1*c*. Results are shown in Figure 2*b* and *c* (i.e., STC B). Link ranking changes significantly when different demand levels are used. As an example, *Link 8* becomes the most critical link, whereas in the previous case it involved no performance loss. This is due to the large increase in demand level associated with *exit 1*, which is directly connected to *link 8* (see Figure 1*a*).

Takeaways: *Critical link ranking is highly variable as different approaches are used. Resilience analysis via topological metrics is limited in the sense that such metrics do not usually take into account traffic demand and network re-configurations following disruptive events. Conversely, the simulation-based stress testing approach is able to capture these aspects thus providing more realistic rankings via the proposed performance loss metric. Stress testing can also be used to compare different road networks and sub-networks by analyzing their response to similar stresses. Travel-time weighted BC produces better estimations of link criticality compared with unweighted BC, which treats all links equally.*

Application to a Real Road Network

To confirm the results of our previous analysis in a realistic scenario, we considered the Paris DIRIF road network, described above. Given the large size of this

(a) DIRIF, geographic location

(b) DIRIF network

	Link	STC A	BC	BC entries-exits	TTWBC	TTWBC entries-exits
Highest-demand links	**95**	117	2628	83	2628	83
	93	102	660	83	660	83
	94	101	1974	83	1974	83
Highest-BC links	**802**	42	192497	3029	192717	3039
	803	27	192521	3029	192741	3039
	607	27	192509	3029	192729	3039
	608	44.6	192449	3029	192669	3039
	397	15.8	83164	1139	83164	1139
	672	14.5	10	1	10	1

(c) DIRIF network link values generated by the different metrics of criticality

Figure 3. The DIRIF road network in Paris agglomeration (*a*) and (*b*). Evaluation results (*c*).

network and the high computation time associated with each network simulation (stress testing one link from such a large network with five capacity drops takes more than one hour on an Intel Xeon E3 CPU equipped with 8 GB of RAM running a Matlab implementation of the mesoscopic simulator, properly configured to handle the DIRIF network in Scenarios A and B), it was prohibitive to perform an exhaustive stress test analysis as in the simple network case. Therefore, we performed stress tests on a limited set of representative links: the three links with the highest demand, the three with the highest BC, and three randomly selected edges with BC in three classes of values (high, medium, and low). We discuss in the following only our simulations related to strategy A. (Simulations for strategy B were in line with the results reported in the previous section and are not discussed due to space limitation.) Figure 3*c* reports the actual values of the considered metrics for the analyzed links. Unlike Figure 1, we do not report metric rankings but actual values of the metrics for each analyzed link. This is motivated by the impossibility of obtaining the full ranking for performance loss (i.e., STC).

Consistently with our previous analysis on the simple network, Figure 3*c* shows that rankings of critical links vary significantly. As an example, *links 95, 93, 94* have a very high value of STC A, whereas the topological metrics rate them much less critical than *links 802, 803, 607*. As pointed out in the previous section, the STC A ranking appears to be more realistic since it captures the higher criticality of *links 95, 93, 94* due to the associated higher demand (not reported due to space limitations).

On the small link subset considered in our analysis, taking into account travel times (TTWBC and TTWBC entries–exits) does not significantly change rankings, since link lengths (and therefore the average travel times) happen to be very similar on all considered links. Finally, it is worth noting that in the DIRIF network BC values (especially in the entries/exits variations) tend to cluster significantly themselves (i.e., many edges have very similar values of BC), thus exhibiting a lower discriminant power than in the case of the simple virtual network.

Takeaways: *In a real-world scenario, stress testing proved to be a realistic and reliable approach to evaluate network resilience. Our evaluation confirms the importance of traffic demand and network dynamics for fine-grained ranking of the most vulnerable road network links. Stress testing has however the drawback of requiring high execution times due to computationally intensive network simulations.*

Discussion and Perspectives

From the previous results, we summarize in the following a few guidelines for properly characterizing critical links with respect to day-to-day disruptions, by means of an intra-network approach in different application contexts.

Firstly, if resilience has to be evaluated in a relatively static context (e.g., network maintenance or planning), BC and TTWBC appear to be adequate. In particular, if data about demand and travel times are not available, we recommend BC, BC from entries and exits, and BC on all paths from entries and exits. These indicators do not require special knowledge of network performance and demand data, but only the basic network topology (links and intersections). If traffic demand is the only missing information, STC, TTWBC, and TTWBC from entries to exits should be preferred, since they also take into account travel time information.

If the goal is instead to achieve a more accurate characterization of network resilience, stress testing should be chosen, since it produces reliable results by taking into account traffic demand and congestion phenomena. The drawback is that it requires many computationally intensive simulations, thus being recommended only in application scenarios that allow for larger computation time, or that address small-sized (sub-)networks. Conversely, in domains with very stringent requirements on response time (e.g., on-line vulnerability monitoring), topological indicators could be the only valuable option. However, it is worth remarking that efficient solutions are still required to compute these metrics on very large networks within reasonable computation time. For future work, we are currently working on the switch from a Matlab implementation of our simulation-based tools to a new implementation based on faster programming languages and approaches explicitly designed for big data processing and real-time computation (e.g., Python and Scala on top of the Spark processing framework). In particular, we believe that implementing real-time advanced solutions for data-driven, on-line monitoring of road traffic resilience will constitute a fundamental research problem to investigate.

We advance that, in order to improve road network resilience analysis, future research work is needed that should consider joining graph-based approaches with demand-aware dynamic stress testing techniques. In this context, we believe that a further improvement with topological metrics could be achieved by modeling the road network as a dynamic graph, whose link weights may change over time depending on actual traffic conditions and both structural and performance-related network properties (e.g., road capacity, real-time traffic information, etc.).

Finally, we argue that future work should also consider area-wide disruptions in addition to single link-based disruptions, especially in the light of measuring the impact of extreme events.

Conclusion

Identifying links critical to the overall network performance is part of road network resilience and intra-network analysis. To this purpose, we have analyzed in this paper several topological metrics based on BC and proposed a stress testing approach exploiting a dynamic simulator. Stress testing appears to be a very promising solution for resilience analysis, allowing for measurement of resilience in terms of the overall performance loss of the whole network consequent to simulated link disruptions.

Our analysis shows that link ranking varies greatly when different metrics are used. As opposed to purely topological metrics, the proposed stress testing approach takes into account demand levels and dynamic characteristics of road traffic. However, it requires more computation time and data than traditional graph-based metrics. The choice of a relevant metric for assessing road network resilience should depend on the context and the specific application requirements.

Merging static topological metrics and demand-based approaches could be of further research interest. It could be relevant to adopt dynamic graphs modeling, using link weights to include dynamic information on the network. In such an approach, topological metrics should be computed dynamically by means of efficient quasi real-time solutions.

Author Contributions

The authors confirm contribution to the paper as follows: study conception and design: Gauthier, Furno, El Faouzi; data collection: Gauthier, Furno, El Faouzi; analysis and interpretation of results: Gauthier, Furno, El Faouzi; draft manuscript preparation: Gauthier, Furno, El Faouzi. All authors reviewed the results and approved the final version of the manuscript.

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