

**ABSTRACT**

Shortest path query over a dynamic road network is a prominent problem for the optimization of real-time traffic systems. Existing solutions rely either on a centralized index system with tremendous pre-computation overhead, or on a distributed graph processing system such as Pregel that requires much synchronization effort. However, the performance of these systems degenerates with frequent route path updates caused by continuous traffic condition change.

In this paper, we build CANDS, a distributed stream processing platform for continuous optimal shortest path queries. It provides an asynchronous solution to answering a large quantity of shortest path queries. It is able to efficiently detect affected paths and adjust their paths in the face of traffic updates. Moreover, the affected paths can be quickly updated to the optimal solutions throughout the whole navigation process. Experimental results demonstrate that the performance for answering shortest path queries by CANDS is two orders of magnitude better than that of GPS, an open-source implementation of Pregel. In addition, CANDS provides fast response to traffic updates to guarantee the optimality of answering shortest path queries.

1. INTRODUCTION

For a modern society, transportation is a major supporting infrastructure that enables the movement of people and goods. With accelerated urbanization worldwide, the number of vehicles on the road and the need for transport are growing rapidly. However, the current transportation systems, with their potential inadequacy at handling fast changing traffic conditions and the optimal control of flows of vehicles, must be rectified to accommodate increasing transportation demands. We can make use of the large amounts of GPS data to optimize the control of transportation systems. But how to effectively and efficiently process the large volume data in a timely fashion posts a substantial challenge for any modern road network system [17].

In recent years, numerous attempts have been made to address these tough problems. For example, Mobile Millennium [18] was proposed as a smart traffic estimation and prediction system. The system processes millions of real-time GPS data using cloud computing and the Spark cluster computing framework. In another case, IBM InfoSphere Streams [11] demonstrates the capability of tackling the challenges of scalability, extensibility and user interaction in the domain of Intelligent Transportation Services [17]. Both Mobile Millennium and IBM InfoSphere Streams provide traffic estimation based on real-time GPS data.

In reality, however, stringent application requirements (such as navigation) and road routing recommendations need to support various kinds of end-users including buses, taxis, ambulances, and fire-engines among others. These users should be notified as soon as possible when a traffic jam has happened on the route they will likely take. They should also be warned and recommended alternative shortest paths. Unfortunately, current systems cannot provide services such as real-time shortest paths. To this end, we aim to tackle two major challenges, by means of providing real-time shortest path services for:

(a) **Exact shortest path query processing in a dynamic graph.** Some existing shortest path approaches [28, 9] adopt a carefully designed index to achieve good performance, but the index based approach is not suited for real-time road networks due to the high cost of index update when edges change. A distributed graph processing system (e.g., Pregel [20], GPS [4] and GraphLab [16]) processes machine learning algorithm on a large graph but does not support a dynamic graph.

(b) **Route adjustment in response to traffic updates.** When an update of road status occurs, e.g., from smooth traffic to congestion, it is a demanding task to find all affected vehicles and update their shortest paths in a real-time fashion. A recent work [21] proposes two approximation techniques in the case of a traffic delay. However, it does not guarantee all the affected vehicles will be notified, nor does it provide optimal shortest paths.

We design and implement CANDS 1, which provides a continuous optimal shortest path service based on a distributed stream processing model. CANDS proposes a distributed solution with an asynchronous mechanism to answer shortest path queries in a dynamic graph. It is a hybrid scheme between index maintenance and distributed graph processing. It maintains shortcuts between each pair of border vertices in a partition. A shortcut is the shortest path between a pair of nonadjacent border vertices within the same graph partition, which is lightweight and easy to maintain in a dynamic environment.

1 The work was done in NUS. Dongxiang Zhang and Jian Cao are contact authors.

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1 The letters come from the title Continuous Optimal Navigation via Distributed Stream Processing.
1. We propose a distributed solution with an asynchronous mechanism to answer shortest path queries in a dynamic graph. Optimization mechanisms are also applied to improve the query processing performance. For example, Message Combine method can merge relevant messages into one message to reduce network traffic; Message Broadcast strategy can quickly limit the traversal region to improve the efficiency. Furthermore, we develop an algorithm to determine the termination of our query processing and prove its correctness.

2. We propose an approach to quickly find affected queries and update their shortest path answers when the road condition changes. Two novel algorithms are designed to incrementally update the affected routes instead of re-computing the shortest path. We prove that the optimality of the shortest path can be guaranteed throughout the whole navigation process. Concurrency control is also discussed when multiple roads are updated at the same time.

3. We develop CANDS on S4 [2], a distributed stream processing system, and deploy CANDS on a cluster. Extensive experiments have been conducted to show the efficiency and robustness of CANDS. Our results show that CANDS processes shortest path queries two orders of magnitude faster than GPS [20].

The remainder of the paper is organized as follows. The problem definition is stated in Section 2. In Section 3, we present the system overview. Sections 4, 5 and 6 describe the details of Graph Initialization, Query Processing and Query Update, respectively. Extensive experimental results are reported in Section 7. In Section 8 existing work is reviewed, and Section 9 concludes the paper.

2. PROBLEM STATEMENT

Table 1: Notations and Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>G</td>
<td>A road network.</td>
</tr>
<tr>
<td>V</td>
<td>Vertices in the road network.</td>
</tr>
<tr>
<td>E</td>
<td>Edges in the road network.</td>
</tr>
<tr>
<td>W</td>
<td>The weights of the edges.</td>
</tr>
<tr>
<td>Q</td>
<td>The shortest path queries.</td>
</tr>
<tr>
<td>G_T</td>
<td>The snapshot of G at timestamp T.</td>
</tr>
<tr>
<td>W_T</td>
<td>The snapshot of W at timestamp T.</td>
</tr>
<tr>
<td>P_{s \rightarrow t}</td>
<td>The shortest path query from starting location s to target location t issued at timestamp T.</td>
</tr>
<tr>
<td>P_{s \rightarrow t}(T)</td>
<td>The result of shortest path for Q_{s \rightarrow t}(T).</td>
</tr>
<tr>
<td>Q_{s \rightarrow t}(T)</td>
<td>All the active navigation queries at timestamp T.</td>
</tr>
</tbody>
</table>

As a convention, we model a road network $G = (V, E, W)$ as a directed weighted graph. Each edge $e \in E$ is a road segment with a certain direction and is represented by $e = (v_i, v_j, w_e)$, where $v_i \in V$ and $v_j \in V$ are road junctions, and $w_e$ is the average travel time to cross the edge. As real-time traffic is considered, we model $G$ as a time-dependent graph and use $G_T = (V, E, W_T)$ to denote the snapshot of $G$ at timestamp $T$.

A vehicle in a road network plays two roles: as a consumer service and as an information provider. It can send a navigation request at any time from anywhere in the network. The request sent at time $T$ is considered a time-dependent single-source shortest path query (SSSP) in $G_T$.

**Definition 1. Time-dependent Shortest Path Query**

Given a time-dependent road network $G_T$ and a pair of source vertex $s$ and target vertex $t$, a shortest path query $Q_{s \rightarrow t}(T)$ returns a path $P_{s \rightarrow t}(T) = v_1 \rightarrow v_2 \rightarrow \ldots \rightarrow v_k$, where $s = v_1$, $t = v_k$, $1 \leq i < k$, and $e_i = (v_i, v_{i+1}, w_e) \in G_T$, such that $\sum_{i=1}^{k-1} w_{e_i}$ is minimized.

A query $Q_{s \rightarrow t}(T)$ is considered active from the timestamp $T$. When the vehicle reaches the destination $t$, the navigation process terminates. We mark the query inactive and remove it from our system.

As an information provider, a vehicle periodically sends its latest context information, including the latitude, longitude, speed and direction, to the back-end servers. There have been several existing works [27, 18] proposed to estimate traffic conditions from vehicle sensor data. In this paper, the estimation component is beyond the focus and we assume that existing solutions are adopted in our system. When a new estimation of a road segment $e$ causes an update on its weight, we need to find which active navigation queries are affected and notify them with better shortest paths. We define this update process as a reverse shortest path query processing for edge update:

**Definition 2. Reverse Shortest Path (RSP) Query for Edge Update**

Suppose at timestamp $T$, the weight of an edge changes from $w_e$ to $w'_e$, and all the active navigation queries are included in $Q_{s \rightarrow t}(T)$. A reverse shortest path query finds all the $Q_{s \rightarrow t}(T)$ which starts from $s$ at $T_1$ and reaches a new location $s'$ at $T$ such that:

1. If $w'_e > w_e$, $e \in P_{s \rightarrow t}(T_1)$, and $e \notin P_{s \rightarrow t}(T)$.
2. If $w'_e < w_e$, $e \notin P_{s \rightarrow t}(T_1)$, and $e \in P_{s \rightarrow t}(T)$.

The first expression identifies queries for which the affected edge, which was part of the shortest path, is no longer so at the current time. The second expression finds queries for which the affected edge, which was not part of the shortest path, is at the current time part of the shortest path.

3. SYSTEM OVERVIEW

The system overview of CANDS deployed on a cluster is shown in Figure 1. It consists of a collection of processing elements (PE) that are customized with specific tasks and allowed to communicate with each other via asynchronous messaging. It is worth noting that this is a basic feature supported in existing distributed stream computing platforms such as Yahoo S4 [2] and Twitter Storm [7] and our system can naturally be implemented on top of these streaming platforms.

CANDS handles two types of streaming inputs: vehicle navigation request and vehicle location update. When a navigation request arrives, CANDS needs to calculate the shortest path from the vehicle’s location to its destination based on the real-time traffic conditions. We propose an asynchronous query processing strategy to reduce query latency and improve system throughput. The other type of event is the periodic GPS reading of the vehicle. These GPS data arrive at CANDS at a high rate, and are used for traffic estimation. When the average travel time of a road segment changes, existing navigation paths passing this road segment may not be optimal and need to be updated with new shortest paths accordingly.

To support optimal navigation, CANDS starts with some initialization steps. It splits the road network into smaller partitions, which are assigned to different nodes in the cluster. The weight of the edge in the road network is the average travel time and some shortcuts are maintained to facilitate query processing. These details of initialization are discussed in Section 4. The functionalities of the PE are also illustrated in Figure 1. It consists of two major components: 1) a query processing engine to handle the time-dependent SSSP queries (details in Section 5) and 2) an incremental update engine responsible for traffic status estimation and sending notifications to affected vehicles (described in Section 6).
4. GRAPH INITIALIZATION
In CANDS, the road network is modeled as a time-dependent graph. The edges are road segments and vertices are road junctions. The weight of each edge graph is estimated as the average travel time to cross the segment. In the following, we present some graph initialization steps before query processing.

4.1 Graph Partitioning
In graph partitioning, the whole road network is first partitioned into M sub-graphs using a METIS-balanced graph partitioning algorithm [19], where M is the number of machines in a cluster. This step is critical to performance improvement. First, the query processing time is normally determined by the slowest task and a balanced partitioning can eliminate the performance bottleneck. Second, communication between vertices in the same sub-graph is done in the same machine and network I/O can be significantly reduced. In the second level, each sub-graph is further split into smaller partitions and each partition is assigned to one PE. Note that this path only guarantees local optimality instead of global one. A better result may be found to contain vertices from other partitions. In this example, $v_1 \rightarrow v_5 \rightarrow v_4$ with distance 4.5 is maintained in $G_1$. The solution proposed in this paper is a hybrid schema between edge-balanced graph partitioning methods can also be applied.

4.2 Graph Shortcut
We maintain a collection of shortcuts to ease query processing. A shortcut is the shortest path of two border vertices within each partition. For any two border vertices $b_1$ and $b_2$, we pre-compute their shortest paths using Dijkstra algorithm and store the results. In the above example, the shortest path $v_1 \rightarrow v_2 \rightarrow v_4$ with distance 4.5 is maintained in $G_1$. Note that this path only guarantees local optimality instead of global one. A better result may be found to contain vertices from other partitions. In this example, $v_1 \rightarrow v_5 \rightarrow v_4$ with distance 4 is the real shortest path from $v_1$ to $v_4$. When an update occurs in the road status in this partition, the shortcuts will be refreshed to ensure the correctness of local optimality.

5. TIME-DEPENDENT SSSP QUERY PROCESSING

Existing solutions on shortest path query processing form two extremes in terms of the index maintenance cost. The index based approaches [10, 12, 28] are very efficient. However, they are difficult to adapt in a dynamic graph due to their prohibitive pre-computation costs. On the other hand, shortest path algorithm is adopted in several distributed graph processing systems [20, 16, 4], and it does not have any index construction cost. But these systems cannot efficiently support shortest path queries in a real-time manner for a huge road network.

The solution proposed in this paper is a hybrid schema between these two extremes. The road network is split into partitions that are assigned to different nodes. In each node, a number of PEs are deployed, each in charge of one partition. In each partition, we maintain some shortcuts between border vertices to facilitate query processing. The shortcuts can be used directly to pass through one partition instead of traversing all the vertices in this partition.

The query processing engine shown in Figure 1 has the following functionalities. EventHandler is responsible for receiving events, emitting events to downstream PE. A termination algorithm (Termination) is designed to determine whether the resultant shortest path is optimal. In each PE, query processing optimization algorithms (Optimization) are further applied to improve the performance.

5.1 Query Processing Algorithm
The query processing strategy, depicted in Figure 3, relies on the coordination of four types of PEs. QueryPE accepts all the navigation requests from vehicles. It stores the graph partition information and knows which partition contains the source vertex of the query. A new RouteEvent is created and sent to ShortPathPE. The event contains eight fields, as listed in Table 2. The gid, s and t are derived from the navigation request. The border node b is initialized to be the same as s and the neighboring partition is initialized.
as the one containing the source vertex. The remaining fields are left empty. When the ShortPathPE receives the message, it starts cooperating with other ShortPathPEs (graph partitions) in an asynchronous manner to find the shortest path. Meanwhile, acknowledgements are sent from ShortPathPE to a StatPE. The StatPE collects the information, determines the termination of the query and sends the shortest path to OutputPE after the termination. The OutputPE then returns the result to the querying vehicle.

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>qid</td>
<td>100000</td>
</tr>
<tr>
<td>source</td>
<td>s</td>
</tr>
<tr>
<td>target</td>
<td>t</td>
</tr>
</tbody>
</table>

Figure 3: PEs handle a shortest path query

Table 2: The fields contained in a Route Event

<table>
<thead>
<tr>
<th>Field</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>qid</td>
<td>query ID uniquely identifies the query.</td>
</tr>
<tr>
<td>s</td>
<td>source vertex of the query.</td>
</tr>
<tr>
<td>t</td>
<td>target vertex of the query.</td>
</tr>
<tr>
<td>b</td>
<td>border node receiving the message.</td>
</tr>
<tr>
<td>nei</td>
<td>neighboring partition which sends the message.</td>
</tr>
<tr>
<td>dist</td>
<td>partial best distance from s to b.</td>
</tr>
<tr>
<td>path</td>
<td>shortest path from s to b.</td>
</tr>
<tr>
<td>ack</td>
<td>acknowledgement sequence from the neighboring partition.</td>
</tr>
</tbody>
</table>

ShortPathPE is the core PE. It cooperates with neighboring partitions to find the shortest path. The idea is similar to Dijkstra algorithm, but without synchronization in each iteration to check which is the best vertex to visit. Our algorithm starts from the graph partition containing the source vertex, denoted by $G_s$. The ShortPathPE spreads the shortest path from $s$ to all the border nodes in $G_p$ to its neighboring partitions. The neighbors receive the events, improve the partial results and further disseminate them to neighbors. Finally, all these partial results will arrive at the graph partition containing the target vertex, denoted by $G_t$. When there is no message propagating in the network, our algorithm can terminate with the correct shortest path, which will be proved in the appendix.

The pseudo-code of event processing in a ShortPathPE with regard to a graph partition $G_p$ is illustrated in Algorithm 1. In the following, we explicitly explain how a ShortPathPE handles an arrival RouteEvent. When an event is received from a neighboring partition, the ShortPathPE needs to check whether the source vertex $s$ or the target vertex $t$ in this event is contained in $G_p$, which leads to four cases:

- **Case 1**: $s \in G_p \land t \in G_p$ (lines 4-12). If both nodes are within $G_p$, we can directly calculate the shortest path from $s$ to $t$ in the subgraph $G_p$. The resultant path is sent to StatPE. However, the algorithm cannot be terminated at this point because the true shortest path may be missed. For example, as shown in Figure 2, the shortest path from $v_1$ to $v_4$ is $v_1 \rightarrow v_5 \rightarrow v_4$ while the Dijkstra algorithm in partition $G_1$ returns $v_1 \rightarrow v_2 \rightarrow v_4$. Therefore, we still need to send the query and partial result to the neighboring partitions even though we have obtained an initial candidate path.

- **Case 2**: $s \in G_p \land t \not\in G_p$ (lines 13-19). If only $s$ is in $G_p$, we call Dijkstra algorithm to calculate the shortest path from $s$ to all the border nodes. Then, all these partial results are sent to neighboring partitions of $G_p$.

- **Case 3**: $s \not\in G_p \land t \not\in G_p$ (lines 20-28). If the current partition is just a bridge between $s$ and $t$, the message is updated by taking account of the shortcut between border nodes and then forwarded to the neighbors. Such a forwarding process is efficient as the shortest path between border nodes has been pre-computed and is available in the local memory. Meanwhile, the partial distance from $s$ to the incoming border node, denoted by $\delta_{s\rightarrow b}(G_p)$, is cached for future pruning. Another advantage is that it can be used to avoid message looping between two neighboring partitions, which generates partial results with loops.

- **Case 4**: $s \not\in G_p \land t \in G_p$ (lines 29-39). If messages containing partial results arrive at the target graph partition, we call Dijkstra algorithm to calculate the path and notify StatPE of the result.

5.2 Query Processing Optimization

Algorithm 1 not only introduces four cases of processing an arrival ShortPathPE, but also incorporates two optimization strategies: Message Combiner and Message Broadcast to improve the processing performance.

5.2.1 Message Combiner

After traversing one partition, messages are sent to neighbor partitions through border edges. For each border edge, one message will be generated and sent to the connecting neighbor partition. It is possible that a partition has multiple border vertices connecting to the same neighborhood. For example, in Figure 2 there are two border vertices $v_1$ and $v_3$ in partition $G_1$. $v_1$ has to send two messages to partition $G_2$ and $v_3$ sends one message to $G_2$ and one message to $G_3$. This message dissemination mechanism incurs too much communication cost in the network. To tackle this problem, we propose a Message Combiner technique to combine some messages together. We add a message combiner before emitting the messages to the neighboring ShortPathPE. If the messages in one partition are sent to the same neighbor partition, they will be merged together as one message (lines 19 and 28 in Algorithm 1).
Algorithm 1 RouteEvent handling algorithm in ShortPathPE

1. \( s \leftarrow \text{event.source}; t \leftarrow \text{event.target}; b \leftarrow \text{event.border}; \)
2. \( \text{dist}(s,b) \leftarrow \text{event.distack} = \text{nextAck}(\text{event.qid}); \)
3. initialize a StatEvents \( sEvent; sEvent.rev[\text{G}_p] \leftarrow 1; \)
4. if \( s \in \text{G}_p \& \& t \in \text{G}_p \) then
5. \( \text{dist}(s,t) \leftarrow \text{Dijkstra}(s,t) \)
6. if \( \text{dist}(s,t) < \delta_{s \rightarrow t} \) then
7. initialize a PathEvent \( pEvent \)
8. \( pEvent.path \leftarrow \text{path}(s,t) \)
9. \( \delta_{s \rightarrow t} \leftarrow \text{dist}(s,t) \)
10. broadcast \( \delta_{s \rightarrow t} \) to all the other ShortPathPE
11. emit \( pEvent \) to ShortPathPE
12. \( T \leftarrow T \cup \{ \text{qid}, \text{G}_p, \text{G}_p, \text{ack} \} \)
13. else if \( s \in \text{G}_p \& \& t \notin \text{G}_p \) then
14. for each \( b' \in \text{L}_n \) .keySet() do
15. run \( \text{Dijkstra}(s,b') \)
16. initialize a \( rEvent \)
17. \( rEvent \leftarrow \{ \text{qid}, \text{dist}(s,b'), b', \text{G}_p, \text{path}(s,b'), \text{ack} \} \)
18. \( \text{combiner}[\text{G}_p][b] \leftarrow rEvent \)
19. \( T \leftarrow \text{CombineByPartition}(\text{combiner}) \)
20. else if \( s \notin \text{G}_p \& \& t \notin \text{G}_p \) then
21. if \( \text{dist}(s,b) < \text{distCache}(s,b) \) then
22. for each pair \( (b', \text{G}_p') \in \text{L}_n \) do
23. \( \text{dist}(s,b') \leftarrow \text{dist}(s,b) + \text{dist}(b,b') \)
24. if \( \text{dist}(s,b') < \delta_{s \rightarrow t} \) then
25. initialize a PathEvent \( pEvent \)
26. \( pEvent.path \leftarrow \text{path}(s,b') \)
27. \( \delta_{s \rightarrow t} \leftarrow \text{dist}(s,b') \)
28. broadcast \( \delta_{s \rightarrow t} \) to all the other ShortPathPE
29. emit \( pEvent \) to ShortPathPE
30. \( T \leftarrow T \cup \{ \text{qid}, \text{G}_p, \text{G}_p, \text{ack} \} \)
31. end if
32. if \( \text{dist}(s,b) < \text{distCache}(s,b) \) then
33. \( \text{dist}(b,t) \leftarrow \text{Dijkstra}(b,t) \)
34. if \( \text{dist}(b,t) < \delta_{s \rightarrow t} \) then
35. initialize a PathEvent \( pEvent \)
36. \( pEvent.path \leftarrow \text{path}(b,t) \)
37. \( \delta_{s \rightarrow t} \leftarrow \text{dist}(s,b) + \text{dist}(b,t) \)
38. broadcast \( \delta_{s \rightarrow t} \) to all the other ShortPathPE
39. emit \( pEvent \) to ShortPathPE
40. \( sEvent.tuples \leftarrow T \)
41. emit \( sEvent \) to StatPathPE

In Figure 2, we combine the RouteEvents between \( G_1 \) and \( G_2 \) as one message and then send it to \( G_2 \).

5.2.2 Message Broadcast

Another optimization technique to avoid message flooding is to broadcast the shortest path to all the ShortPathPEs whenever a better result is found at the target partition \( G_t \). In this way, each partition \( G_t \) maintains a variable \( \delta_{s \rightarrow t} \), indicating the current best distance. The purpose is that the partitions far away from \( s \) can learn this information before receiving messages from neighbors. Then, the message propagation process can stop earlier without involving too many partitions. We define the distance from \( s \) to a graph partition as follows:

\[
d(s,G_p) = \min_{v \in \text{G}_p, \text{border}} d(s,v) \tag{1}
\]

If \( d(s,G_p) > \delta_{s \rightarrow t} \), this propagation can stop. Although this condition may not be satisfied as the message transmission in the network is asynchronous, it can avoid spreading partial worse results from the source partition \( G_s \) to every other partition. The partition far away from \( s \) can dismiss the events if the partial result is found to be worse than the broadcasted result (lines 10 and 36 in Algorithm 1). Although the broadcast overhead is not cheap (the number of messages is the same as that of graph partitions), it still gains an advantage compared to the overhead caused by the communication between any two neighboring ShortPathPEs far from \( s \).

If a graph partition \( G_p \) does not receive any RouteEvent from its neighbors, we have \( d(s,G_p) \geq d(s,t) \). This means this partition is far away from source vertex \( s \) and the shortest path between \( s \) and \( t \) must not pass through this partition. The correctness and feasibility are given in Appendix A.1, A.2, A.3. This property can facilitate the pruning procedure in answering a RSP query in Section 6.

5.3 Algorithm Termination Mechanism

Algorithm termination is a critical issue in distributed query processing. For example, Pregel terminates when every vertex votes to halt in each superstep. In this paper, a solution is proposed for CANDS to determine algorithm termination. We create a special PE, named \( \text{StatPE} \), to receive statistics of messages sent and received (which are encapsulated in \( \text{StatEvent} \)) from ShortPathPE and determine when query processing can be terminated. A \( \text{StatEvent} \) contains multiple tuples in the form of \( (\text{qid}, \text{G}_p, \text{G}_p, \text{ack}) \), each acknowledges the communication with its neighboring partition.

It means the message is sent from \( G_p \) to \( G_p' \) with acknowledgement sequence \( \text{ack} \). Two tuples \( T_1 \) and \( T_2 \) match each other if the following condition is satisfied:

\[
T_1 G_p = T_2 G_p \land T_1 G_p' = T_2 G_p' \land T_1 \text{ack} = T_2 \text{ack}
\]

In \( \text{StatPE} \), we maintain a buffer and a match counter for each query. When a \( \text{StatEvent} \) arrives, we scan the tuples in the event. For each tuple, we scan the buffer to find if there is a match. If a match is found, the tuple is removed from the buffer and the match counter increases by 1. Otherwise, we insert the tuple into the buffer. The algorithm is shown in Algorithm 2. It can be terminated as long as the buffer is empty and \( \text{match} > 0 \). All PEs can fully utilize the CPU resources without being idle. The algorithm can be terminated immediately when the buffer is empty and \( \text{match} > 0 \). There is no need to vote for halt. We prove the correctness of our algorithm in Appendix A.4, A.5, A.6, A.7. Note that when there is message loss, the termination condition will not be satisfied. To avoid endless waiting, we set a threshold for maximum waiting time for early termination of a failed query processing.

Algorithm 2 StatEvent handling algorithm in StatPE

1. for each tuple \( T \) in \( \text{StatEvent} \) do
2. for each tuple \( T' \) in \( \text{buf} \) w.r.t to \( \text{G}_s \rightarrow t \) do
3. if \( T \) matches \( T' \) then
4. remove \( T' \) from \( \text{buf} \)
5. \( \text{match} \leftarrow \text{match} + 1 \)
6. if \( \text{buf} \) is empty and \( \text{match} > 0 \) then
7. emit result to \( \text{OutputPE} \)

6. REVERSE SHORTEST PATH QUERY PROCESSING

In this section we present the incremental update component shown in Figure 1. The traffic conditions are estimated in real-time based on the sensor data from vehicles. Whenever the status of a road changes, the shortcuts calculated previously in the corresponding partition needs to be updated, and a RSP query process is
triggered to find affected vehicles. If a better shortest path is found, the vehicles will be notified instantly. Since a shortest path can be affected by multiple updates at the same time, concurrency control is also required to avoid writing conflicts on the local caches that may lead to a wrong result.

6.1 Shortcut Update Algorithm

When the state of a road segment $e$ at partition $G_e$ changes, we need to update the shortcuts maintained for border nodes in $G_e$. A naive solution is to re-calculate the shortest path for each pair of border nodes, which is quite expensive. To reduce the maintenance overhead of shortcuts, different strategies are adopted in terms of the weight change and whether $e$ appears in the original shortest path between border nodes $b$ and $b'$.

- **Case 1**: $w_e \uparrow \land e \in P_{b \rightarrow b'}$ (lines 4-6). The shortest path between $b$ and $b'$ may not be optimal and we call Dijkstra algorithm to re-compute the shortest path.
- **Case 2**: $w_e \uparrow \land e \notin P_{b \rightarrow b'}$. The original shortcut is not affected by the update and is still optimal. No action is required.
- **Case 3**: $w_e \downarrow \land e \in P_{b \rightarrow b'}$ (lines 8-9). The shortcut between $b$ and $b'$ is still optimal. We simply need to update its distance because $w_e$ decreases.
- **Case 4**: $w_e \downarrow \land e \notin P_{b \rightarrow b'}$ (lines 10-11). In this case, it is possible for border nodes $b$ and $b'$ to find a better result which passes edge $e$. Thus, Dijkstra algorithm is called to guarantee the shortcut is optimal.

6.2 RSP Query Processing

When a RSP query is triggered, we know that the state of an edge $e$ changes. Our goal is to find which active queries are affected. A baseline solution adopts a similar strategy to the shortcut update algorithm that takes into account four cases. It re-computes the shortest path for each pair of border nodes directly from $P_{b \rightarrow b'}$. Similar to the baseline algorithm, we check whether the weight of $e$ increases or decreases and whether it appears in the shortest path from $s'$ to $t$. The query processing algorithm for case 2 and case 3 is the same as that in Section 6.1. In the following, we focus on how to efficiently handle cases 1 and 4.

6.2.1 Case 1: $w_e \uparrow \land e \in P_{s' \rightarrow t}$

To determine whether a navigation query $Q_{s' \rightarrow t}$ with current location $s'$ is affected when $w_e$ increases and $e \in P_{s' \rightarrow t}$, we propose a backward message propagation method. The purpose of this traversal is to find a shortest path that does not pass through edge $e$ and is shorter than existing $P_{s' \rightarrow t}$. If not, the existing path is still the shortest path. An UpdateEvent, which augments the fields in RouteEvent with updated edge $e$ and current location $s'$, is sent to the target partition $G_t$. The sketch of event processing is illustrated in Algorithm 3. When $G_t$ receives the update event, it checks whether $s'$ is contained in $G_t$. If so, Dijkstra algorithm is called to re-compute the shortest path (lines 3-7). Otherwise, the reverse traversal can start at $t$ and pass through border node $h \in G_t$ to other partitions. The distance $\text{dist}(t, b)$ is calculated by Dijkstra algorithm. If $e \notin P_{s' \rightarrow t}$ and $\text{dist}(s', b) + \text{dist}(t, b)$ is smaller than $\delta_{s' \rightarrow t}$, this is a potential shortest path (lines 9-14). If $e \in \text{path}(s', b)$, the border $b$ continues to traverse in a reverse order and propagates it backward to its neighboring partitions (line 16).

In summary, our algorithm traverses in a reverse order from the target partition $G_t$ if the cached path contains edge $e$. This algorithm only traverses the affected regions of edge $e$. The affected regions of edge $e$ are the areas containing the vertex, whose shortest path passes through edge $e$ from source vertex $s$. It stops propagating messages if it does not pass through edge $e$. We can find that the message number is related to the number of affected regions of this edge $e$. We take full advantage of cached paths and distances in the border to reduce the message transmissions. Our algorithm can quickly terminate when there is no message transmission in the network, as proved in Appendix A.8. Therefore, compared to the query processing algorithm that re-computes the shortest path, our approach incurs much less communication overhead.

<table>
<thead>
<tr>
<th>Algorithm 3 BackwardMsgPropagation (UpdateEvent event)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. if $t \in G_p$, then</td>
</tr>
<tr>
<td>2. if $s \in G_p$ then</td>
</tr>
<tr>
<td>3. $\text{dist}(s', t) \leftarrow \text{Dijkstra}(s')$</td>
</tr>
<tr>
<td>4. if $\text{dist}(s', t) &lt; \delta_{s' \rightarrow t}$, then</td>
</tr>
<tr>
<td>5. $\delta_{s' \rightarrow t} \leftarrow \text{dist}(s', t)$</td>
</tr>
<tr>
<td>6. report $\delta_{s' \rightarrow t}$ to StatPE</td>
</tr>
<tr>
<td>7. broadcast $\delta_{s' \rightarrow t}$ to all ShortPathPE</td>
</tr>
<tr>
<td>8. else</td>
</tr>
<tr>
<td>9. if path$(s', b)$ not pass $e$ then</td>
</tr>
<tr>
<td>10. $\text{dist}(s', t) \leftarrow \text{path}(s', b) + \text{dist}(t, b)$</td>
</tr>
<tr>
<td>11. if $\text{dist}(s', t) &lt; \delta_{s' \rightarrow t}$, then</td>
</tr>
<tr>
<td>12. $\delta_{s' \rightarrow t} \leftarrow \text{dist}(s', t)$</td>
</tr>
<tr>
<td>13. report $\delta_{s' \rightarrow t}$ to StatPE</td>
</tr>
<tr>
<td>14. broadcast $\delta_{s' \rightarrow t}$ to all ShortPathPE</td>
</tr>
<tr>
<td>15. else</td>
</tr>
<tr>
<td>16. propagate $\text{event}$ backward to the previous partition along path$(s', b)$</td>
</tr>
</tbody>
</table>

6.2.2 Case 4: $w_e \downarrow \land e \notin P_{s' \rightarrow t}$

When the weight of edge $e$ decreases and $e$ does not appear in the shortest path $P_{s' \rightarrow t}$, we propose a different solution from case 1 to detect the affected queries. The purpose of this traversal is to find a shortest path that passes through edge $e$ and is shorter than existing $P_{s' \rightarrow t}$. If not, the existing path is still the shortest path. The algorithm handling case 4 is based on forward propagation and shown in Algorithm 4. Initially, an UpdateEvent is initialized and sent to the partition $G_e$ containing $e$ whose weight is updated. We traverse all cached queries in $G_e$. If a query $Q_{s', t}$ is the initial source vertex and current location of that vehicle is $s'$, we check whether the weight of $e$ increases or decreases and whether it appears in the shortest path from $s'$ to $t$. The query processing algorithm for case 2 and case 3 is the same as that in Section 6.1. In the following, we focus on how to efficiently handle cases 1 and 4.

\footnote{The cache contains path $P_{s \rightarrow b}$ and we can derive path $P_{s' \rightarrow b}$ from $P_{s \rightarrow b}$.}
Algorithm 4 ForwardMsgPropagation($UpdateEvent\text{~event}$)

1. for Query $Q_{r\rightarrow}$ cached in $G_e$ do
2. if $r \in G_e$ then
3. if $s \in G_e$ then
4. $b \leftarrow s'$
5. else
6. $b$ is the incoming border node
7. dist($b, t$) ← Dijkstra($s, t$)
8. if dist($s, b') + $dist($b, t$) < $δ_{s'\rightarrow}$ then
9. $δ_{s'\rightarrow}$ ← dist($s, t$)
10. report $δ_{s'\rightarrow}$ to StatPE
11. broadcast $δ_{s'\rightarrow}$ to all the ShortPathPE
12. else
13. for each outgoing border node $b'$ for query $Q_{s\rightarrow}$ do
14. if $e$ is contained in the updated shortcut $(b, b')$ then
15. for Query $Q_{r\rightarrow}$ which passes through $e$ do
16. dist($s', b'$) ← dist($s', b$) + $dist(b, b')$
17. emit a RouteEvent with partial result dist($s', b'$) to neighboring partition through border node $b'$

outgoing border nodes $b'$ to the neighboring partition. If the updated shortcut between $b$ and $b'$ passes through $e$, we create a new RouteEvent with the better distance dist($s, b'$) and emit the event to ShortPathPE in the query processing. The following processing of the new generated RouteEvent is the same as that in Section 5 and omitted here. The algorithm will terminate when there is no message in transit. Finally, we prove that Algorithm 4 finds all the affected queries in case 4 and can correctly update their shortest paths in Appendix A.9.

6.3 Concurrency Control

In CANDS, when there is a weight update in an edge, a reverse shortest path query is triggered and all the affected queries will be updated. Since an active navigation query could be affected by multiple weight updates at the same time, concurrency control mechanism should be adopted to guarantee the correctness of the result. Figure 4 shows a write conflict example. Suppose initially at timestamp $T_1$, the shortest path from $A$ to $D$ is $A \rightarrow B \rightarrow C \rightarrow D$ with distance 12. At $T_2$ ($T_2 > T_1$), the first update occurs and the weight of $AB$ increases from 3 to 5. Then, a reverse shortest path query $U_1$ is triggered to update all the affected navigation queries. However, before $U_1$ finishes, another edge update occurs and the weight of $BC$ decreases from 4 to 2 at timestamp $T_3$ ($T_3 > T_2$). It triggers another query processing $U_2$ for edge update of $BC$. It is possible that $U_2$ finishes earlier than $U_1$ and update the distance $δ_{A\rightarrow D}$ maintained in each processing element to be 10. Finally, at timestamp $T_4$ ($T_4 > T_3$), $U_1$ finishes. However, it does not take into account the update of $U_2$ and $δ_{A\rightarrow D}$ is updated to 14. But the real shortest distance from $A$ to $D$ should be 12 at timestamp $T_3$. To solve the problem, we use a simple lock mechanism to guarantee that the query processing for edge update is executed sequentially. More specifically, when there is an edge update, we first check whether the lock is occupied by other update events. If yes, the event is pushed into a waiting queue. Otherwise, it gets the lock, processes the edge update event and releases the lock. In this manner, we can avoid a navigation query being affected by multiple concurrent edge updates. It is worth noting that all our experiments are conducted with such locking mechanism.

7. EXPERIMENTS

7.1 Experiments with Synthetic Workloads

We implement CANDS on S4 [2], which is a general-purpose distributed stream processing system under Apache incubation. S4 provides friendly programming interfaces and supports an unbounded stream. This system is event driven and query processing in each PE can be triggered periodically or by an incoming event. The version of Apache S4 is 0.6.0. It is worth noting that although we have deployed CANDS on S4, CANSD can also be implemented on other stream systems such as Twitter Storm [7]. All of our experiments are run on a cluster with ten nodes. Each node has one Xeon E5607 Quad Core CPU (2.27GHz), 32GB memory, running CentOS 6.2. We select one node for Zookeeper and data source adapter in the customization of S4.

We study the performance of SSSP and RSP query processing in CANDS. For SSSP query, we report the results of our asynchronous algorithms without optimization (Async), improved with Message Broadcast only (Async+MB), improved with Message Combiner only (Async+MC), and using both optimization techniques (Async+MB+MC). We compare performances of CANDS and GPS (Graph Processing System), an open source implementation of Google’s Pregel. For RSP query processing, we compare with a baseline algorithm which re-computes an SSSP query if the query is detected to be affected by the status update of an edge. We measure the performance of SSSP and RSP query processing from the following four aspects:

**Average query latency.** The latency of an SSSP query is the time interval between the time it arrives at the system and the time it has been processed. For RSP query, the latency incorporates the time waiting for the release of the lock of concurrency control and the query processing time, which refers to the time for detecting all the affected queries and updating them with better routes. If a query fails, we do not count it when calculating the average query latency.

**Throughput.** Throughput is measured by the number of queries processed in a time unit. In our setting, the time unit is one minute.

**Network I/O.** The network I/O is measured by the amount of bytes of messages transferred in the network, which is the same metrics to measure the communication cost in GPS.

**Success rate.** Since messages may be lost when the message arrival rate is too high, we use the query success rate as a measurement of system availability. For SSSP query, it is the number of successful queries divided by the total number of queries. For RSP query, we consider it successful only if all the affected navigation requests are updated with new correct results. Note that this measures the success rate for first time processing and we have fault tolerance for failed query (e.g., re-submit the query).

In the following, we report performance evaluation using both synthetic and real workloads, and all experiments are conducted with concurrency control described in Section 6.3.
7.1.1 Datasets and Experiment Setups

Our experiments use datasets derived from US road network [3]. We pick 5 representative road networks of different sizes and summarize them in Table 3. The smallest dataset contains only 320K vertices and the largest one contains more than 14 million vertices. By default, we use the CAL dataset with moderate size to test the performance.

**Table 3: Road Network Datasets**

<table>
<thead>
<tr>
<th>Name</th>
<th>Region</th>
<th>Vertex Number</th>
<th>Edge Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAY</td>
<td>San Francisco Bay Area</td>
<td>321,270</td>
<td>800,172</td>
</tr>
<tr>
<td>FLA</td>
<td>Florida</td>
<td>1,070,376</td>
<td>2,712,798</td>
</tr>
<tr>
<td>CAL</td>
<td>California and Nevada</td>
<td>1,890,815</td>
<td>4,657,742</td>
</tr>
<tr>
<td>E</td>
<td>Eastern USA</td>
<td>3,598,623</td>
<td>8,778,114</td>
</tr>
<tr>
<td>C-IR</td>
<td>Central USA</td>
<td>14,081,816</td>
<td>14,292,498</td>
</tr>
</tbody>
</table>

We evaluate the performance in terms of increasing numbers of nodes in a cluster, length of path route, number of graph partitions, frequency of edge updates and number of active queries. We also test the hybrid case where SSSP and RSP queries arrive together to indicate the ratio of RSP queries to the total number of query events. The parameters are listed in Table 4, with the default setting in bold format.

**Table 4: Experiment Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>cluster nodes</td>
<td>1, 2, 3, 4, 5, 6, 7, 8, 9</td>
</tr>
<tr>
<td>length of shortest path</td>
<td>[1,100], [100,200], [200,300], [300,400], [400,500], [500,600]</td>
</tr>
<tr>
<td>graph partition</td>
<td>10, 100, 500, 1000, 5000, 10000</td>
</tr>
<tr>
<td>update frequency</td>
<td>200, 400, 600, 800</td>
</tr>
<tr>
<td>active SSSP queries</td>
<td>10, 100, 500, 1000, 5000, 10000</td>
</tr>
<tr>
<td>ratio of RSP query</td>
<td>0.1, 0.3, 0.5, 0.7, 0.9</td>
</tr>
</tbody>
</table>

7.1.2 SSSP Query Processing

We first examine the performance of SSSP query in terms of increasing number of nodes in a cluster. The query latency, throughput, network I/O and success rate are reported in Figure 5. We compare with GPS and report the performance with different optimization techniques. When more nodes are used, it incurs more network I/O cost to process SSSP queries. However, the query latency and system throughput are both improved because we have more memory and CPU resources for parallel computing. The message combiner (MC) plays a more important role than message broadcast (MB) in the optimization of SSSP query processing. It is very effective in reducing network I/O and query latency. It improves system throughput by 5 times. MB method can reduce the communication cost, as shown in Figure 5(c), but the effect is limited.

GPS uses bulk synchronous parallel model and in each superstep, a vertex handles messages coming from its neighboring vertices. If there is no incoming message in the current iteration, it votes to halt. Therefore, given a query with length \( L \) (meaning the route has \( L \) edges), it requires at least \( L \) supersteps to finish a query. In Figure 5(a), the length of SSSP query results is between 100 and 200 and we report the performance of 100 supersteps in GPS. When deployed in 9 nodes, our system takes only 132ms to answer a SSSP query and supports more than 1000 queries in one minute without any failure. The query latency of our system is two orders of magnitude better than GPS.

The performance of SSSP query processing is also sensitive to the query result length. We increase the length from [1, 100] to [500, 600] and show the performance in Figure 6. The query latency grows with increasing path length. By comparing the query latency between Async+MC and Async+MC+MB, we can see that message combiner is more effective in performance improvement when the length increases. Async+MB has 100% success rate if the length is less than 200 and decreases when more than 200. The success rates of Async+MC and Async+MB+MC are still very high even if the path length of SSSP query is very large (e.g., 500-600). Async+MB is more affected than Async+MC and Async+MB+MC by path length.

7.1.3 RSP Query Processing

The query latency and network I/O of SSSP query processing with increasing number of graph partitions are estimated in Figure 7. The communication I/O grows significantly when the number of partitions increases from 10 to 10,000. The query processing time first drops when the graph is split into more partitions. When the partition number is very small, Dijkstra algorithm is expensive to find the distance from source vertex \( s \) to border vertices or from border vertices to target vertex \( t \). However, when the partition number grows to thousands, the performance degrades because it incurs high communication cost and much effort is spent in sending and receiving messages. Async+MB+MC always achieves the best performance as the partition number grows.

In the final experiment of SSSP query processing, we report the average query latency and success rate in different road networks in Figure 8. When the graph size increases, the PEs need to take charge of a larger graph partition and the performance of SSSP query processing degrades. In CTR, which contains 14 million vertices, the elapsed time for one query is more than 700ms. The success rate also drops when the graph size increases dramatically. But our system still achieves low query latency and high success rate in a larger graph.
We compare the efficiency of query update between RSP query and re-computing query. Re-computing query represents a baseline approach that resubmits the query to find the new shortest path once the query is affected. The performance of RSP query processing is shown in Figure 9. The default number of active queries is 1000. When the number of nodes increases, the query latency drops slightly. The successful rate is always 100 percent. With 9 nodes, the average time to run an RSP query is 273ms. Our method utilizing the cached partial results achieves two orders of magnitude of performance improvement over the baseline algorithm which uses re-computation to update the affected queries.

The update frequency is another aspect to estimate the performance of our system. We apply three methods on choosing update edges. Random randomly chooses some edges to update with random values. SmallPeak selects update edges based on Random method, but the weights of some edges are updated with high values. Partition-Peak means some partitions are very busy and the edges in these partitions are peak. RSP queries can be finished in one second and 100 percent success rate when the number of partition is more than 500. In Figure 12, we estimate the performance of these three methods and our system can quickly process a series of weight update.

We report the average query latency and success rate with increasing number of active SSSP queries in Figure 10. With more active queries in the system, more of their shortest paths will be affected by an update of the road state. Therefore, it takes more time to complete an RSP query. When the number of active queries increases to 5000, it takes 4 seconds to detect and update affected queries and the success rate also drops significantly.

The update frequency is another aspect to estimate the performance of our system. When the weights of many edges are updated at the same time, a lot of RSP queries may be affected. In Figure 11 we present the average query latency and success rate with increasing update frequency. Our RSP queries can finish in one second when update frequency is less than 500 per minute. The result shows that RSP processing can keep up with increasing number of edge updates.

Since a traffic jam might affect many edges, the partition will be very busy to process related edges. It will affect the performance of our system. We apply three methods on choosing update edges. Random randomly chooses some edges to update with random values. SmallPeak selects update edges based on Random method, but the weights of some edges are updated with high values. Partition-Peak means some partitions are very busy and the edges in these partitions are peak. RSP queries can be finished in one second and 100 percent success rate when the number of partition is more than 500. In Figure 12, we estimate the performance of these three methods and our system can quickly process a series of weight update.

The performance of RSP query processing on all five datasets is shown in Figure 13. If the size of road network is small, our method is very efficient with 100 percent success rate. It takes 36ms to process an RSP query in the BAY dataset. Even in the CTR road network which contains more than 14 million roads, the average response time to an RSP event is around 700ms. There is no query failure in these datasets except CTR. The result shows that RSP query can finish the query update in a short time even in a large road network.
7.2 Hybrid Query Event Processing

We control the ratio of RSP queries in a hybrid scenario where SSSP and RSP queries arrive in a mixed way. The ratio of RSP queries indicates the ratio of RSP queries among the total number of query events. The total number of queries is 300 and the ratio increases from 0.1 to 0.9. The average latency of SSSP query processing improves when the ratio becomes larger. As we can see from Figure 14, the query latency of RSP query processing increases as the ratio becomes larger. The reason is the concurrency control mechanism works when an SSSP query is affected by multiple road weight updates. Thus, it takes more waiting time when there are more RSP queries in the event stream. The success rate of query processing is nearly 100 percent when the ratio varies.

![Figure 13: Performance in different road networks](image)

![Figure 14: Performance w.r.t. to ratio of RSP query](image)

7.3 Experiments with Real Workloads

The real trace dataset contains the GPS trajectories of 10,357 taxis during the period of Feb. 2 to Feb. 8, 2008 within Beijing [27]. There are 154,662 vertices and 337,662 road segments in the Beijing road network. Each trajectory is a sequence of GPS records with timestamp and status information. These GPS records are mapped to the corresponding network edges and the original travelling path is inferred [25]. We split a day into multiple time intervals and for each interval, we estimate the weight of road network by the average travel time of all the taxis passing through the edge in that period. We extract the boarding and alighting points of passengers as \( s \) and \( t \) in query \( Q_{s \rightarrow t} \). In this way, we generated 200 queries. The trace of these queries has an average of 200 path lengths.

The performance of SSSP query is evaluated with increasing partition number and node number. The query latency and success rate are reported in Figure 15. The best partition number is 100 and has higher success rate. The query latency of Async+MC and Async+MB+MC decreases with more nodes in all query methods, which is similar to that for Synthetic Workloads. The success rates of Async and Async+MB are higher than those for Synthetic Workloads.

![Figure 15: Performance Evaluation of SSSP query](image)

![Figure 16: Performance Evaluation of RSP query](image)

8. RELATED WORK

Shortest path query processing has been well studied [23, 14, 8, 12, 22, 28]. Most of the recent works focus on developing index
structures to support query processing in a huge road network. For example, ALT [13] selects some vertices as landmarks and precomputes the distance to these landmarks to accelerate query processing. RE [14] adds shortcuts between pairs of selected vertices to help pruning. In TNR [10], grid network is adopted to reduce the search space. In [8, 12, 22, 28], the vertices are organized in a hierarchical structure and shortcuts between different levels of vertices are added. Although these indexes are efficient in answering shortest path queries, they require tremendous pre-computing cost to build the index and are not suitable for a dynamic road network.

There are various mining algorithms proposed to predict travel time across a road segment. For example, T-drive [26] mines smart driving directions from the historical GPS trajectories of a large number of taxis and constructs a time-dependent landmark graph to recommend the best route. Another adaptive algorithm [15] is proposed to estimate traffic condition from historical traffic data. VTrack [24] can estimate the travel time along the route based on the noisy information from different sources such as WiFi. IBM also implemented a real-time traffic estimation system based on IBM InfoSphere Streams [11, 5]. MobileMillenium [18], developed in Spark [6], is another system supporting traffic estimation and prediction. It infers traffic conditions using GPS measurements from drivers running cell phone applications, taxicabs, and other mobile and static data sources.

Distributed graph processing systems such as Pregel [20], GPS [4] and Giraph [1] are developed to efficiently process large-scale web graphs and various social networks. These systems require much effort in synchronization and are designed for general graph processing algorithms. Therefore, our customized method can achieve two orders of magnitude improved performance. Although GraphLab [16] also adopts asynchronous methods to support scalable graph mining in an asynchronous manner, its current implementation only works in a static graph.

In response to traffic delay update, Malviya [21] proposed two approximate techniques, K-paths and proximity measure, to monitor a set of designated travel routes. Our system has two advantages compared to this work. We return exact results and notify all the affected vehicles when there is a traffic update.

9. CONCLUSION

In this paper, we studied the problem of the shortest path over a dynamic road network and implemented the service on a distributed stream processing platform S4. More specifically, we proposed an asynchronous and optimized method to efficiently compute shortest path over a dynamic graph. To guarantee the optimality in the navigation process, we monitor the traffic updates in real-time and notify affected vehicles if a better route is found. Experiments on real road networks with synthetic traffic loads and real trajectory trace verified the efficiency of our system. It takes less than 150ms to respond to a navigation request in a network with millions of vertices if the route length is moderate. It also supports 10,000 active navigation queries without failure when identifying affected queries by a status update of any road segment. Our system can achieve low query latency asynchronous query processing and fast response time on query updates.

10. ACKNOWLEDGEMENTS

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11. REFERENCES

A. THE PROOF OF LEMMA

Here we prove that the cached distance in any partition \( G_p \) is more than the shortest path \( d(s,t) \) if this partition does not receive any RouteEvent.

**Lemma A.1.** For any partition \( G_p \), its cached \( \delta_{s \rightarrow t} \geq d(s,t) \).

**Proof.** If the partition does not receive a broadcast message from the destination PE, \( \delta_{s \rightarrow t} \) is initialized to \( \infty \) and thus \( \delta_{s \rightarrow t} \geq d(s,t) \). Otherwise, we have found a path from \( s \) to \( t \) with distance \( \delta_{s \rightarrow t} \). Since \( d(s,t) \) is the distance of the shortest path, \( \delta_{s \rightarrow t} \geq d(s,t) \).

**Lemma A.2.** For any partition \( G_p \), if \( d(s,G_p) < d(s,t) \), \( G_p \) receives at least one RouteEvent from its neighbors.

**Proof.** Assume that the shortest path from \( s \) to \( G_p \) is \( s \rightarrow \ldots \rightarrow u \rightarrow v \) where \( u \in G_p \), \( u \notin G_p \), and \( v \in G_p \). In other words, \( G_p \) is the neighboring partition of \( G_p \) with a connecting edge \((u,v)\). We know that in \( G_p \), we have \( \text{dist}(s,u) + \text{dist}(u,v) = \text{dist}(s,G_p) < d(s,t) \). From Lemma A.1, we have \( d(s,t) \leq \delta_{s \rightarrow t} \). Therefore, \( \text{dist}(s,u) + \text{dist}(u,v) < \delta_{s \rightarrow t} \) and a RouteEvent will be sent from \( G_p \) to \( G_p \) based on Algorithm 1.

**Lemma A.3.** If a graph partition \( G_p \) does not receive a message w.r.t. to \( Q_{s \rightarrow t} \) from its neighbor, we have \( d(s,G_p) \geq d(s,t) \).

**Proof.** It is easy to prove from Lemma A.2.

Now we prove that if \( \text{match} > 0 \) and \( \text{buf} = \emptyset \), the algorithm can be terminated with correct shortest path.

**Lemma A.4.** Given a message propagation chain from \( G_1 \rightarrow G_2 \rightarrow \ldots \rightarrow G_p \), if a set of acknowledgements from \( \{G_1, G_2, \ldots \} \) arrives earlier than \( G_i \) where \( j_k > i \), we can find at least one acknowledgement from \( G_{j_k} \) \((j > i)\) pending in the buffer.

**Proof.** Let \( j_k = \min \{G_{j_1}, G_{j_2}, \ldots \} \). Since the acknowledgement of \( G_{j_k} \) has not arrived, the acknowledgement from \( G_{j_k} \), \( \text{qid}_{G_{j_k}}, \text{G}_{j_k}, \text{ack} \) cannot find a match and will be inserted into the buffer as in Algorithm 2.

**Lemma A.5.** If \( \text{match} > 0 \) and \( \text{buf} = \emptyset \), all the RouteEvents have been processed.

**Proof.** Suppose at time point \( t \), we have \( \text{match} > 0 \) and \( \text{buf} = \emptyset \) in StatPE and there still exists one RouteEvent sending to \( G_p \) to \( G_{p} \). That event has not been processed. The event could be either under transmission in the network or queuing in the event buffer in \( G_p \).

We assume that the partial result from \( s \) to \( G_p \) in this message is \( s \rightarrow v_1 \rightarrow v_2 \rightarrow \ldots \rightarrow u \rightarrow v \) and this path crosses a set of partitions \( G_1 \rightarrow G_2 \rightarrow \ldots \rightarrow G_{j_k} \). If there exists no partition \( G_i \) from which StatPE has received a tuple \( \text{qid}_{G_i}, \text{G}_{i+1}, \text{ack} \) used to acknowledge its sending a RouteEvent to the neighbor \( G_{i+1} \) in this path. According to Lemma A.4, the buffer will not be empty; If StatPE has received an acknowledgement from \( G_i \). Suppose \( G_i \) is closest to \( G_{i+1} \) in this chain and the acknowledgement \( G_i \) has been sent to StatPE. Since the acknowledgement from \( G_{i+1} \) has not arrived, the one from \( G_i \) cannot be matched and it will be pending in the buffer. This contradicts the fact that the buffer is empty.

**Lemma A.6.** If \( \text{match} > 0 \) and \( \text{buf} = \emptyset \) in StatPE, the algorithm can terminate with the correct shortest path from \( s \) to \( t \).

**Proof.** From Lemma A.5, we know that when \( \text{match} > 0 \) and the message buffer in StatPE is empty, all the RouteEvents have been processed and acknowledged and there is no need for the StatPE to continue waiting for messages coming from ShortPathPE. The algorithm can terminate.

Let \( P \) be the real shortest path and \( P = (s \rightarrow v_2 \ldots \rightarrow (v_{11} \rightarrow v_{12} \ldots) \rightarrow (v_{21} \rightarrow v_{22} \ldots) \rightarrow \ldots \rightarrow (v_{m1} \rightarrow \ldots \rightarrow v_{mt}) \) which passes a sequence of \( m \) graph partitions \( (G_1) \rightarrow (G_2) \rightarrow \ldots \rightarrow (G_m) \). Support a node \( v_i \) on the shortest path, that is \( d(s,t) = d(s,v_i) + d(v_i,t) \). If there exists a new path from \( s \) to \( v_i \) where \( d(s,v_i) < d(s,t) \). It will send the new path with the partial result to neighbor partitions which leads to a contradiction as there is no message in the network. Therefore, our algorithm finds the optimal shortest path from \( s \) to \( t \).

Similar to Lemma A.6, we can prove the correctness of the following lemma:

**Lemma A.7.** When the algorithm terminates, for any border node \( b \), if \( \text{dist}(s,b) < \delta_{s \rightarrow t} \), then \( \text{dist}(s,b) \) is the shortest distance from \( s \) to \( b \).

**Proof.** Here we prove Algorithm 3 and Algorithm 4 can efficiently handle the edge update on Case 1 \((w_{e} \uparrow \wedge e \in P_{u \rightarrow v})\) and Case 4 \((w_{e} \downarrow \wedge e \notin P_{u \rightarrow v})\) respectively.

**Lemma A.8.** Algorithm 3 on Case 1 is correct.

**Proof.** Suppose \( P = \{s \rightarrow u_1 \rightarrow \ldots \rightarrow u_t \rightarrow v_{t+1} \rightarrow \ldots \rightarrow t \} \) is the old shortest path passing the update edge \( e = (u_t, u_{t+1}) \) and \( P' = \{s \rightarrow v_1 \rightarrow v_2 \rightarrow \ldots \rightarrow t \} \) is a new shortest path with a smaller distance. It is obvious that \( e \) does not appear in the new path \( P' \). If \( s_1 \) \( v_1 \) and \( t \) are in the same partition, the shortest path will be returned by calling Dijkstra algorithm (lines 3-7 in Algorithm 3). Otherwise, we split \( P' \) into two segments \( P'' = (s' \rightarrow \ldots \rightarrow v_i) \rightarrow (v_{i+1} \rightarrow \ldots \rightarrow t) \), where \( v_i \) is the first node in the backward direction of path \( P' \) such that \( P_{u \rightarrow v} \) in the cache of ShortPathPE does not contain edge \( e \). Otherwise, all the \( v_i \) in path \( P' \) whose cached path \( P_{u \rightarrow v} \) contains edge \( e \). It means \( e \in P' \), which is a contradiction. According to Lemma A.7, since the cached \( dist(s',b) < \delta_{s \rightarrow t} \), we know that \( dist(s',b) \) is the shortest distance from \( s' \) to \( b \). Since the cached partial results from \( s' \) to \( v_{i+1} \), \( v_{i+2} \), \ldots , \( t \) contain edge \( e \), the partition containing \( v_i \) will receive a backward propagation (line 16 in our algorithm). Thus, it will find a better result by merging the cached shortest path from \( s' \) to \( v_i \) and the propagated partial result from neighboring partition (lines 9-14).

**Lemma A.9.** Algorithm 4 on Case 4 is correct.

**Proof.** Suppose \( P = \{s' \rightarrow v_1 \rightarrow \ldots \rightarrow v_i \rightarrow v_{i+1} \rightarrow \ldots \rightarrow t \} \) is a new shortest path and \( \{v_{j_1} \rightarrow v_{j_2} \rightarrow \ldots \rightarrow v_{j_t} \} \rightarrow \ldots \rightarrow t \) is a new shortest path and \( \{v_{j_1} \rightarrow v_{j_2} \rightarrow \ldots \rightarrow v_{j_t} \} \rightarrow \ldots \rightarrow t \) is a new shortest path and \( \{v_{j_1} \rightarrow v_{j_2} \rightarrow \ldots \rightarrow v_{j_t} \} \rightarrow \ldots \rightarrow t \) is a new shortest path and \( \{v_{j_1} \rightarrow v_{j_2} \rightarrow \ldots \rightarrow v_{j_t} \} \rightarrow \ldots \rightarrow t \) is a new shortest path and \( \{v_{j_1} \rightarrow v_{j_2} \rightarrow \ldots \rightarrow v_{j_t} \} \rightarrow \ldots \rightarrow t \) is a new shortest path and \( \{v_{j_1} \rightarrow v_{j_2} \rightarrow \ldots \rightarrow v_{j_t} \} \rightarrow \ldots \rightarrow t \) is a new shortest path and \( \{v_{j_1} \rightarrow v_{j_2} \rightarrow \ldots \rightarrow v_{j_t} \} \rightarrow \ldots \rightarrow t \) is a new shortest path. If \( t \notin G_k \), suppose \( v_{j_{t+1}} \) is the outgoing border node of \( G_k \) in path \( P' \). Since the shortcut \( (v_j, v_{j+1}) \) passes the update edge \( e \), we create a RouteEvent which contains the partial result \( path(s', v_{j+1}) \) and propagate it to the neighboring partitions. Similar to the proof in Lemma A.6, we can show that \( P' \) will be found by our query processing algorithm.