Hervé Rivano

Habilitation à Diriger les Recherches

Linear Programming techniques for modeling capacity and energy issues in wireless multi hop networks

23 juin 2014
Amphi Chappe - Laboratoire CITI - INSA Lyon

Me, Myself, and others

Followed by

- Capacity of wireless networks
- Modeling interferences
- Solving models
- Introducing energy
- What's the point anyway?

Last ten years

2012 : Creation of UrbaNet team
2011 : Chargé de Recherche Inria
2009 : Moving to the CITI lab (Swing team)
2004 : Chargé de Recherche CNRS, I3S lab (Mascotte team)
2003 : 1/2 ATER IUT Génie Télécom Réseau UNSA
2003 : PhD thesis University of Nice Sophia Antipolis - FT R&D
« *Algorithms and telecommunications : Approximate coloring and multi-commodity flow applied to infrastructure networks* »
Advised by Afonso Ferreira and Jérôme Galtier

Responsibilities :

- Leader of Inria/INSA Lyon team UrbaNet since 2012
- Steering committee of ResCom since 2010
- CITI lab council since 2009
- Section 7 of Comité National de la Recherche Scientifique, 2008-2012
- I3S lab council, 2007-2009

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Academic collaborations

Collaboration contexts:

- Equip-Ex Sense-City: experimental urban playground for sensors
- Labex IMU: « Urban Smart Worlds », with social and economics scientists
- CityLab@Inria project lab: Inria teams addressing SmartCities issues

People and teams:

- Catherine Rosenberg (U. Waterloo)
- Nathalie Mitton (Equipe Inria Lille FUN)
- Marcelo Dias De Amorim (LIP 6)
- Alfredo Goldman (U. Sao Paulo)
- Coati (ex Mascotte, Inria Sophia Antipolis),
- Drakkar (INPG)

- Paris XIII (Khaled Boussetta)
- CNR (Marco Fiore)

Grants and partnership management

Management of research grants:

- IMU PrivaMov project 2013-2016, leaded by LIRIS, 200k€, CITI coordinator
- ANR IDEFIX 2013-2016, leaded by Orange labs, 1M€, Inria coordinator
- ARC Région « mobilité et services urbains » 2012-2015, 100k€, leader
- BQR INSA Lyon ARBRE 2012-2014, 30k€, leader
- ANR Ecoscells 2009-2012, leaded by Alcatel-Lucent, 1M€, Inria coordinator
- ARC Inria CARMA 2007-2008, 100k€, leader
- ANR Jeune Chercheur OSERA 2005-2007, 100k€, leader

Industrial partnerships:

- Alcatel-Lucent
- Orange Labs
- 3Roam (PME Sophia Antipolis)

Teaching and scientific mediation

Academic teaching

- Optimization: M2 MDFI U. Marseille 2008, M2 RTS U. Lyon 1 2010 (2*12h)
- Network and network administration: IUT GTR 2003-2004 (92h)
- Approximation algorithms: DEA RSD UNSA 2000-2008 (10h/y)
- Programming (java, Scheme, C): UNSA 2001-2003 (30h/y)
- Computer science primers: CNAM de Nice 1999 (30h)

Scientific mediation

- « *Networks for digital cities* », tutorial for ISN teachers, 2014
- Organization of « *Digital Cities Days* », Insa Lyon, 2013
- « *Linear programming, relaxation, column generation* », Optimization school of Alcatel-Lucent/Inria common lab, 2012
- « *Network primers* », ENS Lyon « semaine ski », 2012
- « *Smart cities issues* », Bubble Spark tutorial, 2011
- Co-organization of « *eHuman in digital society* » colloquium, French pavillon of universal exposition, Shanghai, 2010

PhD advising

Defended thesis

- Anis Ouni, ANR Ecoscells (2009-2013), postdoc Telecom ParisTech
  - « *Optimisation de la capacité et de la consommation énergétique dans les réseaux maillés sans fil* »
- Christelle Molle-Caillouet, DGA/CNRS (2006-2009), MCF UNSA Coati
  - « *Optimisation de la capacité des réseaux radio maillés* »
- Patricio Reyes, Conicyt/Inria (2005-2009), postdoc U. Madrid
  - « *Collecte d'Information dans les Réseaux Radio* »

Ongoing thesis

- Soukaina Cherkaoui, ADR Green Alcatel-Lucent/Inria (2013-)
  - « *Economies d'énergie dans les réseaux cellulaires hétérogènes* »
- Trista Lin, ARC région, avec Dynamid et UrbaLyon (2012-) :
  - « *Mesure de la mobilité humaine pour une cartographie des services* »

Capacity of wireless networks

Followed by

- Modeling interferences
- Solving models
- Introducing energy
- What's the point anyway?

Looking back 10 years ago

2004 : routing and wavelength assignment in optical networks

- Core networks
  - Operator viewpoint
  - Network design and provisioning issues
- Approximation algorithm methodology
  - Provide guarantees on approximation ratio and complexity
  - Graph theoretic and algorithmic culture

The rise of mobile communication

- Promising technology for under-equipped regions
  - « Internet in the villages » project
- Feeling of being useful for fast expanding urban zones

« How to compute the capacity of a multi-hop wireless network? »

A choice of methodology: optimization

Several other methods in the literature

- Performance evaluation and simulation
  - Average results, sometimes higher order statistics
  - Evaluate one strategy at a time
- Distributed algorithms and combinatorics
  - Modeling with evolving graphs [A. Ferreira, Networks 2004]
  - Algorithmic complexity and lower bounding results, theoretical models

Lack of understanding extremal properties of networks

Use of operational research and linear programing results for

- Modeling networks fundamental properties
- Compute optimal configurations
- Analyse the underlying structures

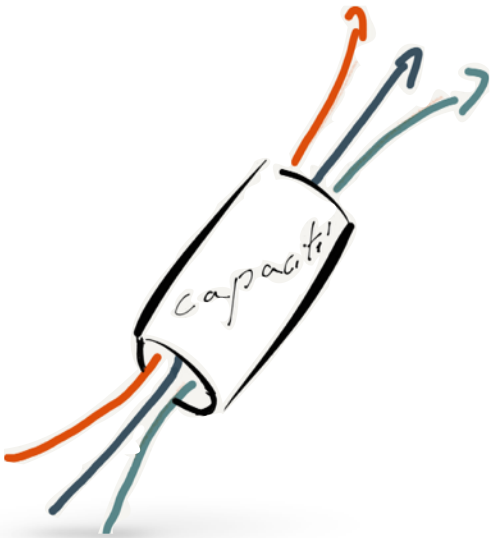
Fundamentals of wired network capacity

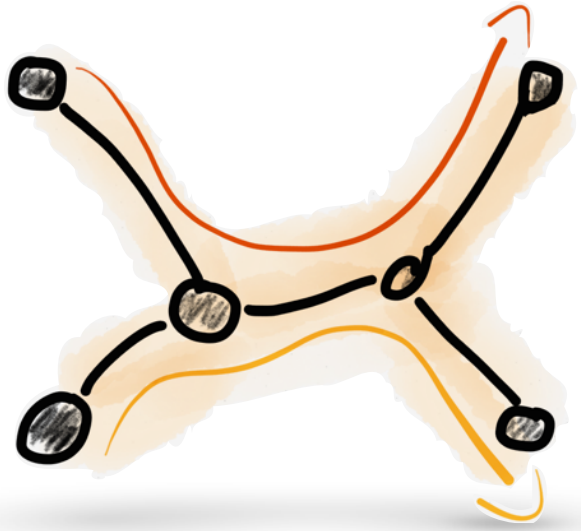
Routing information flows and sharing local ressources on each link

- Perfect medium, queuing policies and saturated mode hypothesis
- Modeling with multi-commodity flows

Assigning global ressources to entities competing over the network

- Binary and absolute competition hypothesis
- Modeling with coloring of conflict graph





Radio is neither local nor global

Signal to Noise Ratio

- Continuous phenomenon

Shannon links capacity and SNR

$$SNR(u,v) = \Theta\left(\frac{\gamma(u,v).P(u)}{\mathcal{N}(v)}\right)$$

$$D(u,v) = \Theta\left(\log\left(1 + SNR(u,v)\right)\right)$$

Transmit Power (W)

Radio is neither local nor global

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Shannon links capacity and SNR

Modulation and coding schemes

- Discretization of capacity
- Tx « ok » threshold

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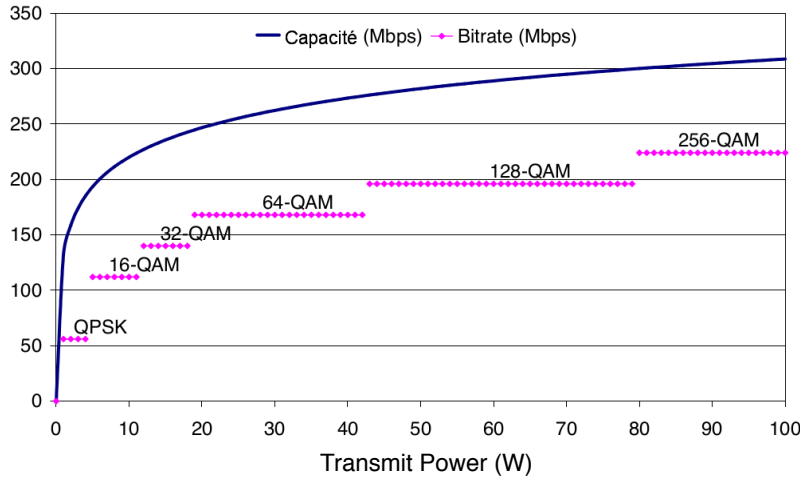

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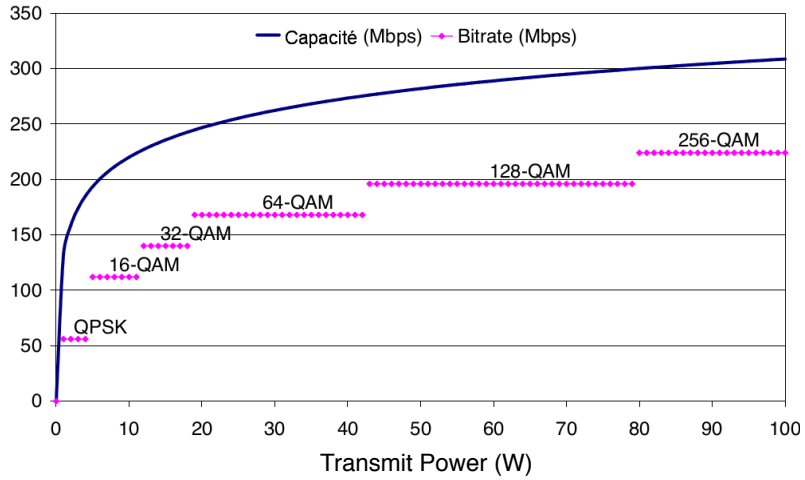


Combinatorial notion of radio link

$$E = \{(u,v) \mid \exists P(u),\ i,\ SNR(u,v) > \sigma_i\}$$

$$c(u,v) = max_i\{c_i \mid \exists P(u),\ i,\ SNR(u,v) > \sigma_i\}$$

Combinatorial modeling of interferences

Signal to Noise and Interference ratio

- Cumulatif effect
- Non binary competition

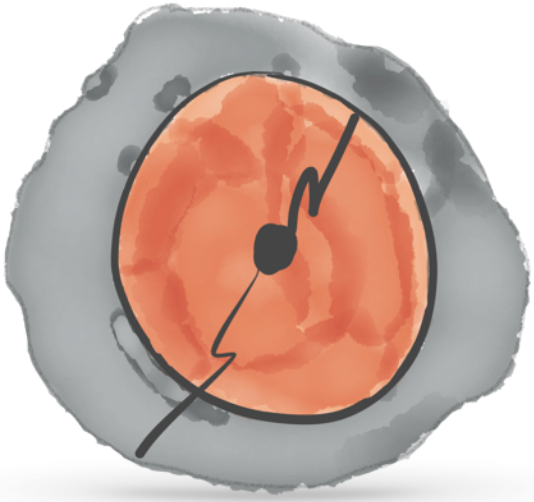
$$SINR(u,v) = \frac{\gamma(u,v)P(u)}{\mathcal{N}(v) + \sum_{w \neq u,v} \gamma(w,v)P(w)} \geq \sigma$$

To define a conflit graph, a strong approximation

- Fixed transmit power
- Perfect communication within range, 0 after
- 100% interference within larger range, 0 after

To alleviate isotropic propagation hypothesis

- Random perturbation of range



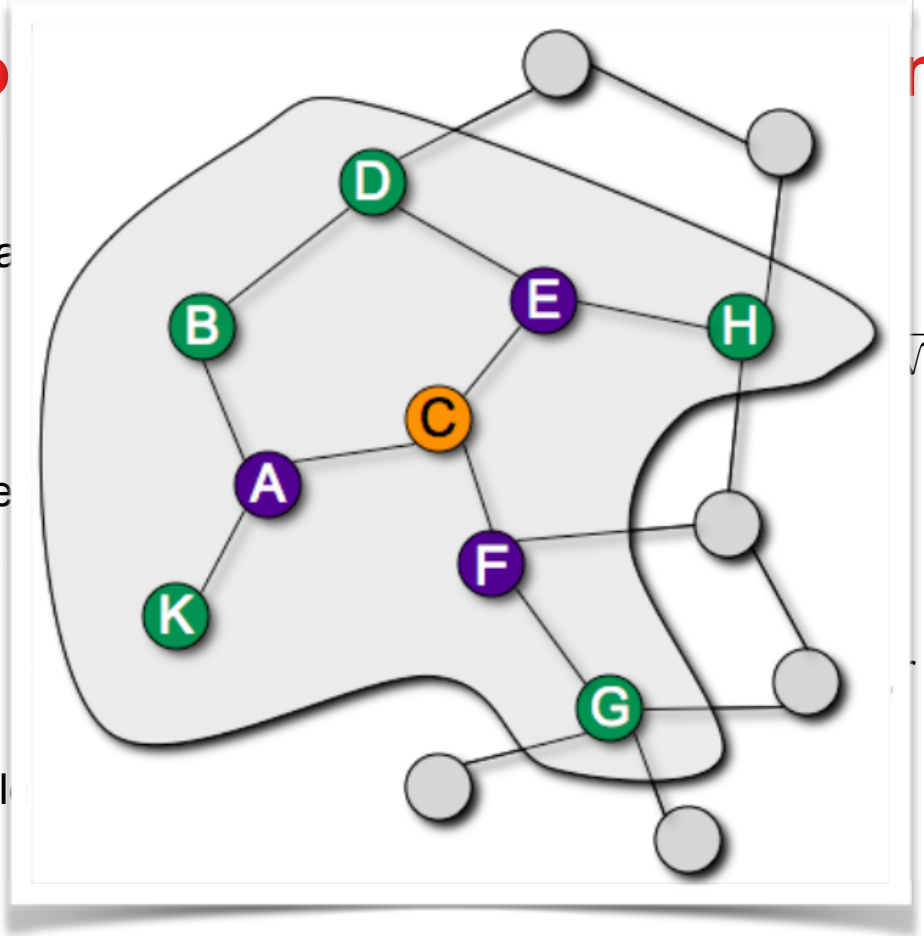
Co nterferences

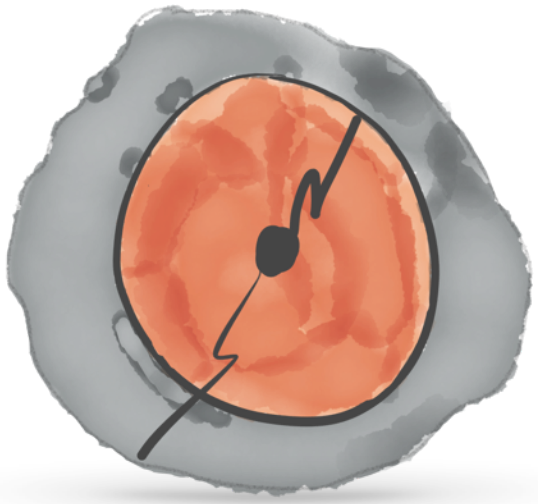
Signa

$$\frac{\gamma(u,v)P(u)}{\int(v) + \sum_{w \neq u,v} \gamma(w,v)P(w)} \geq \sigma$$

To de

To all





2-hop interference model

- Contention with nodes at 2 hops
- **Capacity sharing inside a neighborhood ?**

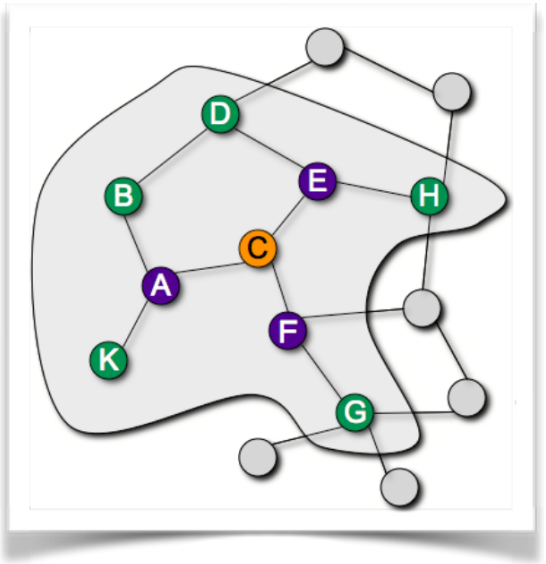
2-hop interference model

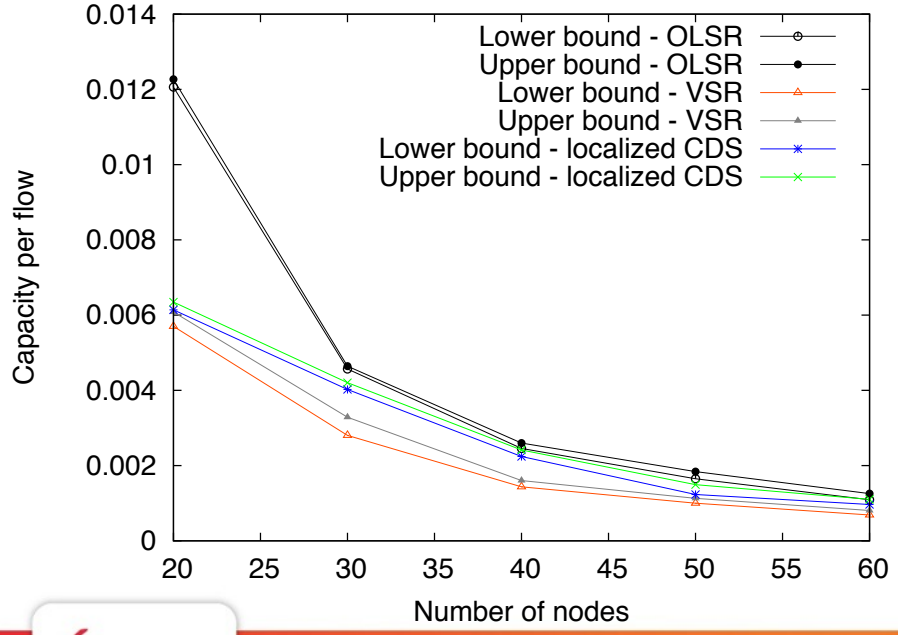
Under and over estimating capacity reuse

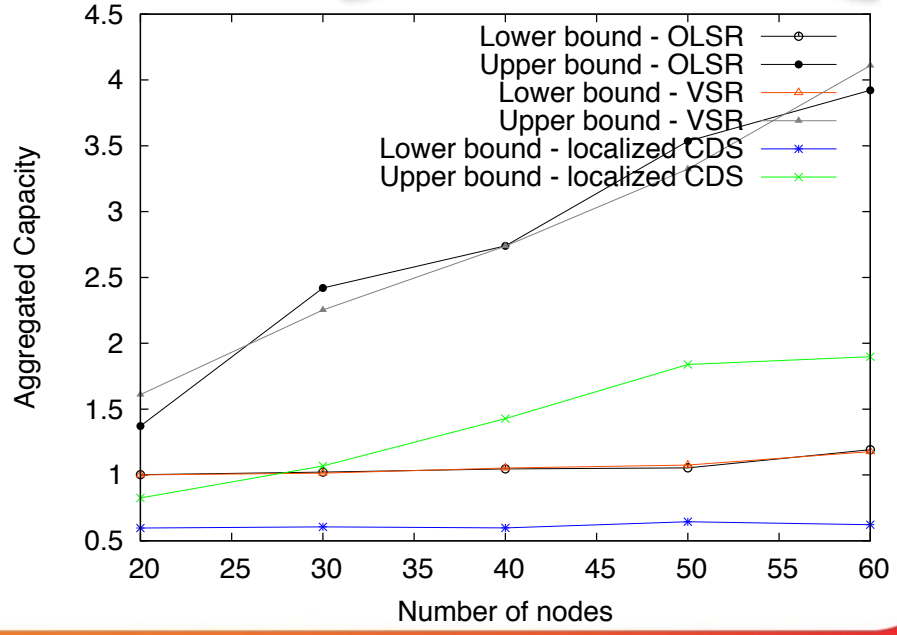
Routing over path computed by simulation

- Gives also signaling overhead

Allows for a « fair » comparison of protocols







Modeling interferences

Followed by

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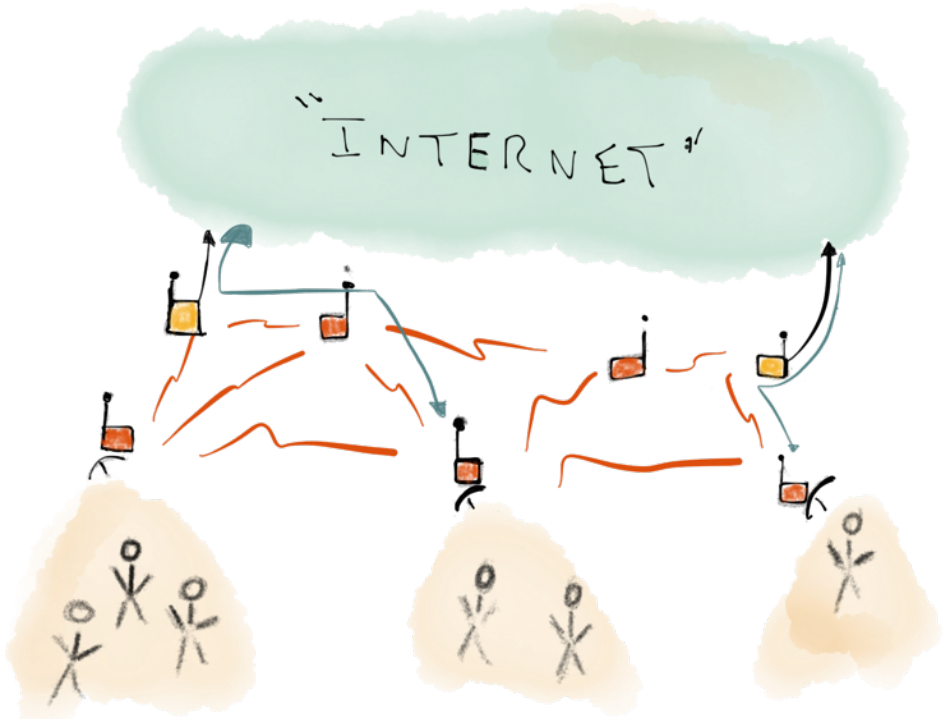
- What's the point anyway?

Wireless mesh networks

Mesh topology

- Aggregated user traffic
- Routers-gateway flows
- Traffic demand: d_r
- Time multiplexing



Steady state hypothesis

- Periodic system
- Period of T (or 1)

Capacity :

- Link : $c(e).\dfrac{\#\text{slots with } e \text{ active}}{T}$
- Network : $\dfrac{\sum_r d_r}{T}$

Routing and scheduling, binary model

$$
\begin{aligned}
\text{Max} \quad & \sum_{r} d_r & & \\
\text{s.t.} \quad & a(t,e) + a(t,e') & \leqslant & \; 1, \; \forall e' \in \mathcal{I}(e), t \leqslant T \\
& \sum_{r \in V} f_r(e) & \leqslant & \sum_{t \leqslant T} a(t,e).c(e), \; \forall e \in E \\
& \sum_{(u,v) \in E} f_r(u,v) & = & \sum_{(u,v) \in E} f_r(v,u), \; \forall r \in V, \forall u \in V \backslash \{r\} \\
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Breaking symmetries

Explicite timing but no impact on objective

- T! equivalent solutions
- Poor linear relaxation

Graph « coloring » and multi-commodity flow combined

- Independent sets = sets of simultaneously actionable links
- Coloring = covering by independent sets

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- Coloring = covering by independent sets

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$$\sum_{P \in \mathcal{P}_r} f(P) \geqslant d_r,\ \forall r \in V$$

Independent sets

Given a fixed routing

- Weighted coloring of links
- Providing link capacity

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Interference model included within the description of the independent sets

- Binary model: stable set of the conflict graph

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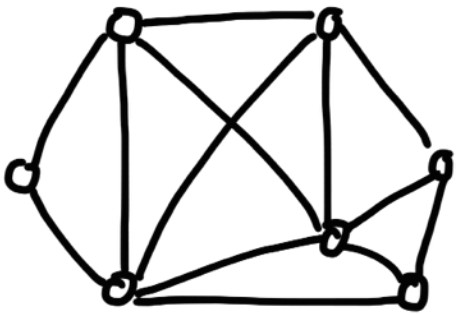

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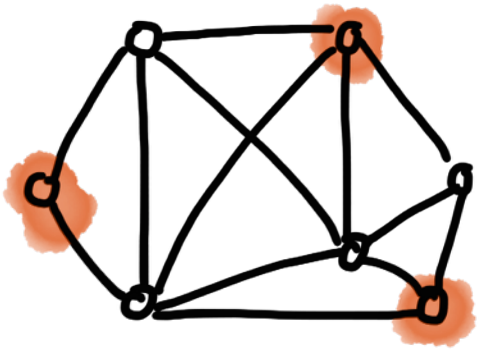

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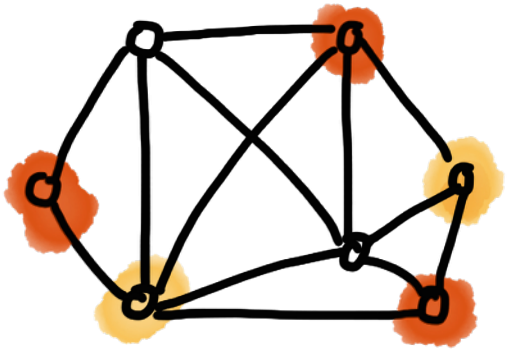

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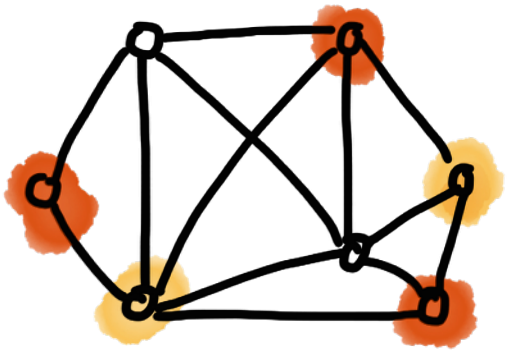
- SINR model is possible



$$\forall (r,v) \in I, \frac{P \cdot \gamma(r,v)}{\mathcal{N}(v) + \sum_{w \neq \{r,v\}} P \cdot \gamma(w,v)} \geqslant \sigma$$

$$\sum_{v \in \Gamma(r)} z(r,v) \leqslant 1, \forall r \in V$$

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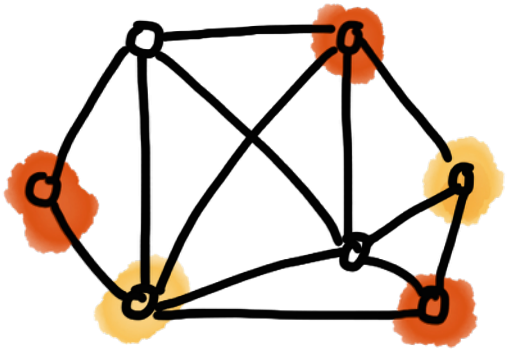

How to manipulate so many variables?

Solving models

Followed by

Introducing energy

What's the point anyway?

Duality

$$Min\ c.x$$
$$Ax \geq b$$

$$Max\ b^t.y$$
$$A^t y \leq c^t$$

Duality

Each LP has a Dual LP

- Dual of Dual is Primal
- Solving Primal gives dual values

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Dual coeff matrix = transposed of primal

- One dual variable per constraint
- One dual constraint per variable
- Too many variables —> too many constraints

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Dual built by feasibility of Primal

- x solution of Primal —> values for y
- x non optimal solution <—> y not a solution
- x non optimal <—> a dual constraint is violated
- x optimal solution <—> y optimal too

$$Min\ c.x$$
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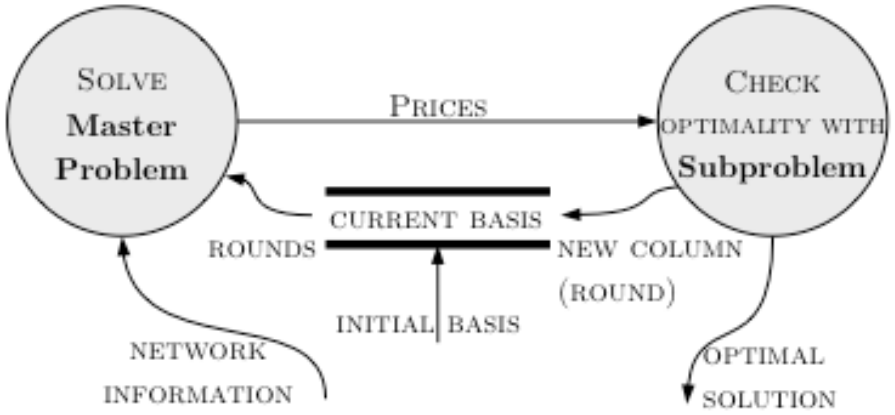
Principle of Column Generation algorithm

Column Generation

$$Min\ c.x$$
$$Ax \geq b$$



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CG Algorithm:
repeat
GET CURRENT BASIS
SOLVE MASTER PROBLEM
CHECK OPTIMALITY
if (not OPTIMAL)
SAVE NEW COLUMN
until (OPTIMAL SOLUTION)

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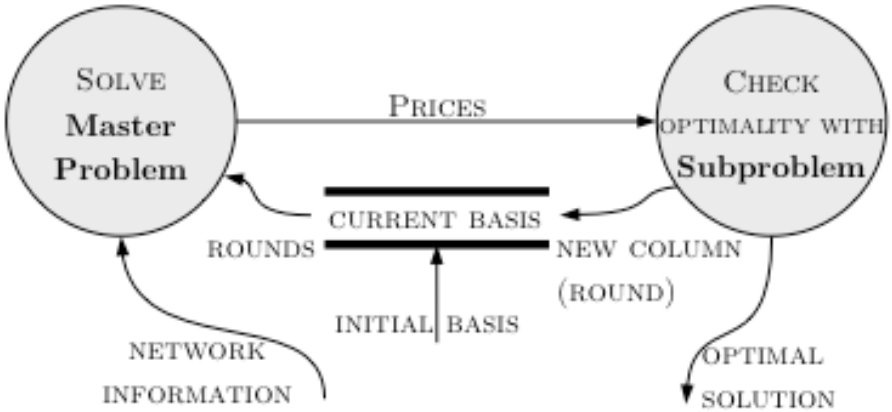

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2 challenges :

- Find a violated dual constraint
- Translate it into a primal variable

Interpretation of the dual

An example with flows

$$Max \sum_{p \in \mathcal{P}} f(p)$$

$$s.t.$$

$$\sum_{p \ni e} f(p) \leq c(e), \ \forall e$$

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Constraint of the Dual : the weight of any path greater than 1.

- In particular the shortest.

Column generation:

- Start with a trivial set of path (yet providing a solution)
- Compute the sub-optimal flow on this restricted basis
  - obtain dual multipliers (lambda)
- Find a shortest path on the lambda-weighted graph
  - If too short, insert it on the primal basis and loop
  - If not, optimality is guaranteed

Capacity of WMN : Joint Routing & Scheduling Problem

$$\text{Min} \quad \sum_{I \in \mathcal{I}} w(I)$$

$$\text{s.t.} \quad \sum_{P \in \mathcal{P},\, P \ni e} f(P) \leqslant c(e). \sum_{I \in \mathcal{I},\, e \in I} w(I),\ \forall e \in E$$

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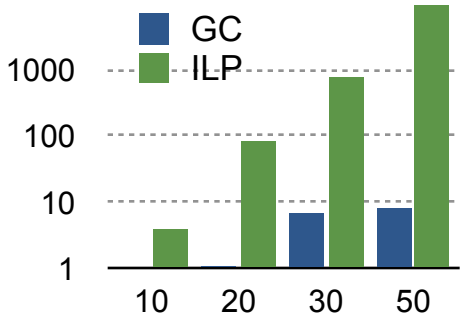

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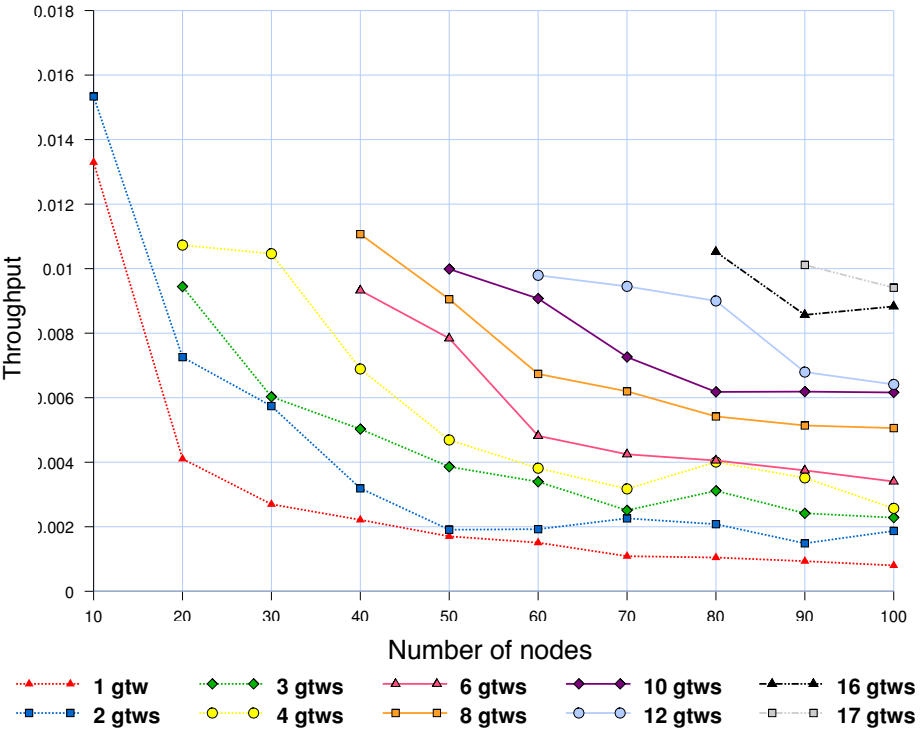
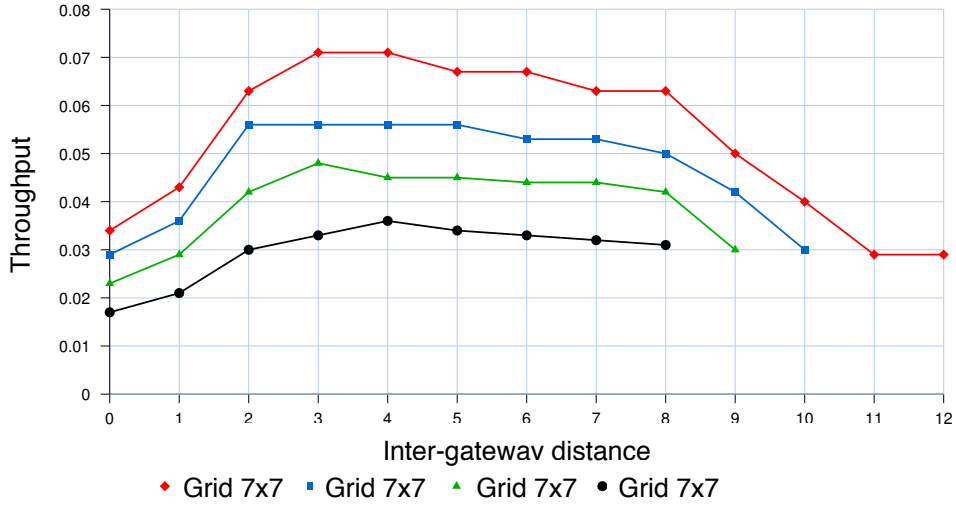
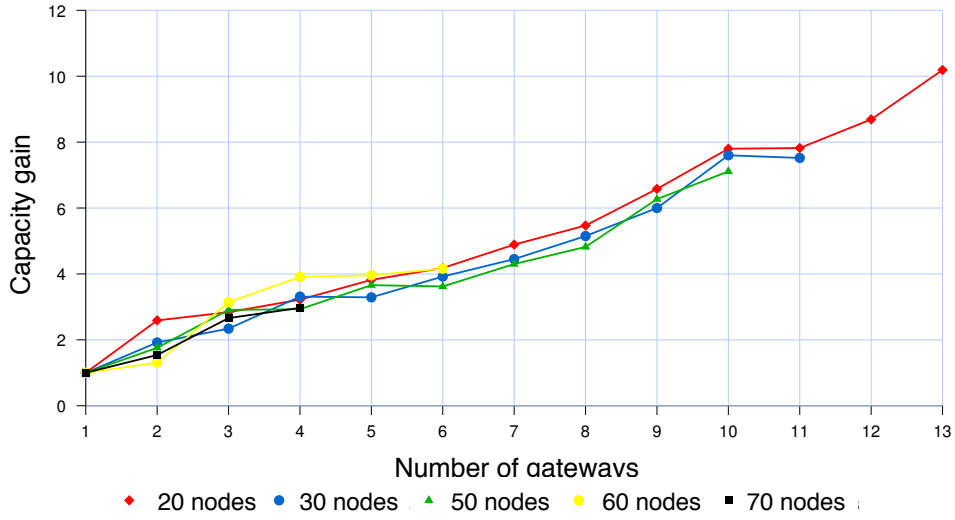
$$\sum_{v \in \Gamma(r)} z(r,v) \leqslant 1, \forall r \in V$$

Global models obtains useful results







Yes ! … but

Understanding some fundamentals of the capacity

- Gathering patterns create bottlenecks
- Area surrounding gateways determine the capacity
- Spatial diversity is crucial

Exploit this structure, speed up computation, almost no limit on network size

- Cuts are a prominent structure
- Non constrained routing far from bottlenecks
- Combined column and line generation

But the model has to be extended

- Uniform and fixed transmit power
- Uniform and unique Modulation and Coding Scheme
- TDMA on one radio channel

Introducing energy

Followed by

What's the point anyway?

Energy is the prominent challenge

Society challenges ICT and telecommunication infrastructures

- Explosion of mobile data consumption
- Fear of radio waves exposure
- Rise of the energetic crisis

Typical Radio Access Networks = 1% of modern countries energy consumption

- « More of the same » = 1 nuclear reactor per country in 10 years
- Overwhelming OpEx for operators

Contradictory *a priori*

- Shannon: « more capacity needs more power »
- Spatial reuse means capacity, needs lower transmit power

Trade-off between energy consumption and network capacity?

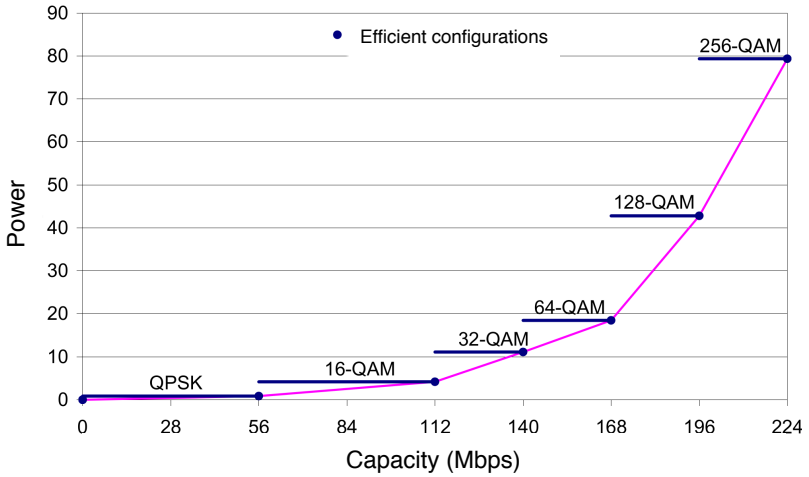
Transmit power control

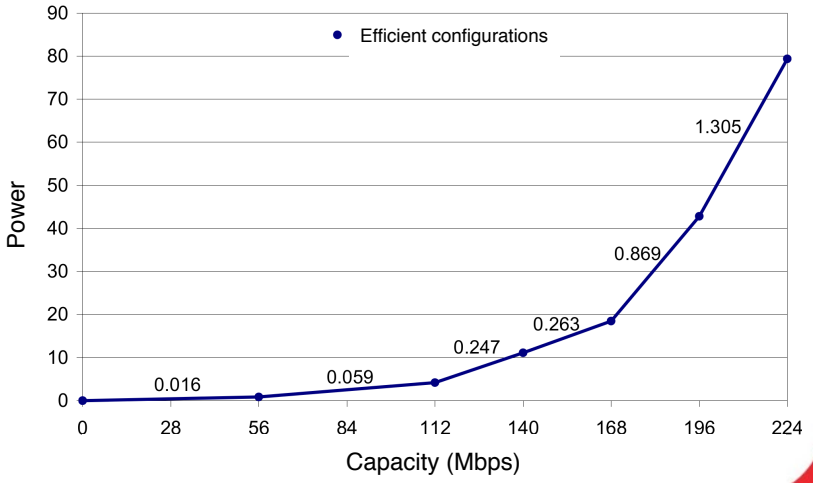
Hypothesis of continuous power control
- Mostly available - at least by steps
- Combinatorial notion of links?

Shannon : power non linear with capacity
- With MCS, discrete step function

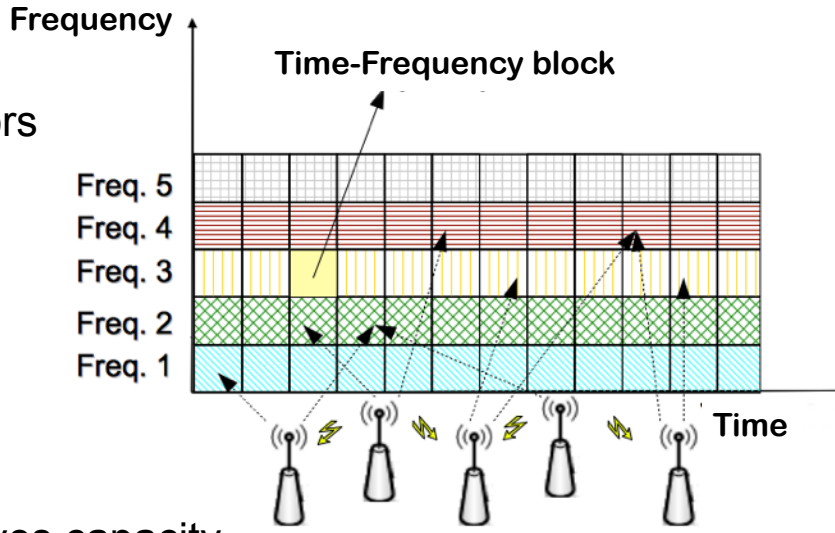
General case very hard to handle
- Buy at bulk scenarios

Hopefully, convex shape
- Piecewise linearization
- Slope increases at each step
- Fill 1st step, then 2nd, …





OFDMA networks



OFDMA available for cellular, mesh, sensors

- Time and Frequency multiplexing
- Scheduling blocks (SB)

Notion of physical link

- Source, destination, channel gain
- Transmit power
- Modulation and coding scheme gives capacity

Connexion *implemented* by a physical link

- Possibly on several SB simultaneously
- Maximum sum of simultaneous transmit powers
- Logical capacity is sum of physical capacities

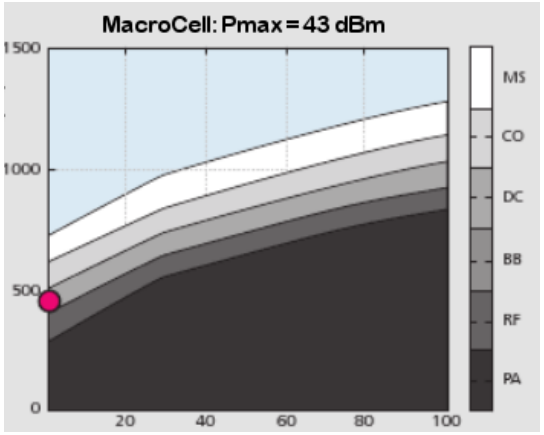
Energy model

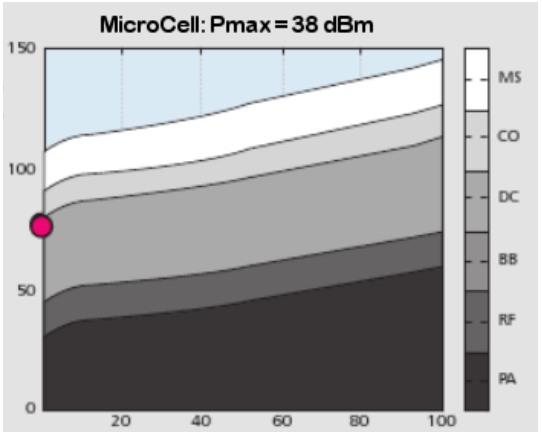
Commonly adopted energy model (from European project Earth)

- Fixed power P0 when idle
- Fixed power P0 + Pr when receiving
- Affine power P0 + aPe when sending at transmit power Pe

Hypothesis of nodes « always on »

- Considering P0=0
- No impact on solution but shift on numerical values





Routing and scheduling with energy budget

$$\text{Min} \quad \sum_{I \in \mathcal{I}} w(I)$$

$$\text{s.t.} \sum_{I \in \mathcal{I}} J(I).w(I) \quad \leq \quad J_{max}$$

$$\forall r \in V, \sum_{P \in \mathcal{P}_r} f(P) \quad \geqslant \quad d_r$$

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Or, minimize energy with capacity budget

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ISet with continuous power control

$$\text{Max} \quad \sum_{\substack{e\in E\\ \kappa\in\mathcal{K}}} \lambda(e).c_\kappa(e) - \eta \sum_{u\in V} J(u)$$

$$\forall u \in V, \quad a(u). \sum_{\kappa\in\mathcal{K}} P_e^\kappa(u) + \sum_{\substack{v\in V\\ \kappa\in\mathcal{K}\\ i\leq M}} P_r(u)\Psi_\kappa^i(v,u) \quad = \quad J(u)$$

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$$\forall u \in V, \kappa \in \mathcal{K}, \qquad \sum_{\substack{v\in V\\ i\leq M}} \Psi_\kappa^i(u,v) + \sum_{\substack{w\in V\\ i\leq M}} \Psi_\kappa^i(w,u) \quad \leq \quad 1$$

$$\forall e \in E, \kappa \in \mathcal{K}, \qquad c_\kappa(e) \quad = \quad \sum_{i\leq M} c_i \Psi^i_{(u,v),k}$$

$$\forall u \in V, \qquad \sum_{\kappa\in\mathcal{K}} P_e^\kappa(u) \quad \leq \quad \bar{P}(u)$$

Not-so-easy to solve

Independent sets with continuous power control, MCS selection

- Lots of binary variables
- Complexity: number of channels and MCS

Still possible to handle average sized networks

- Can be accelerated a bit

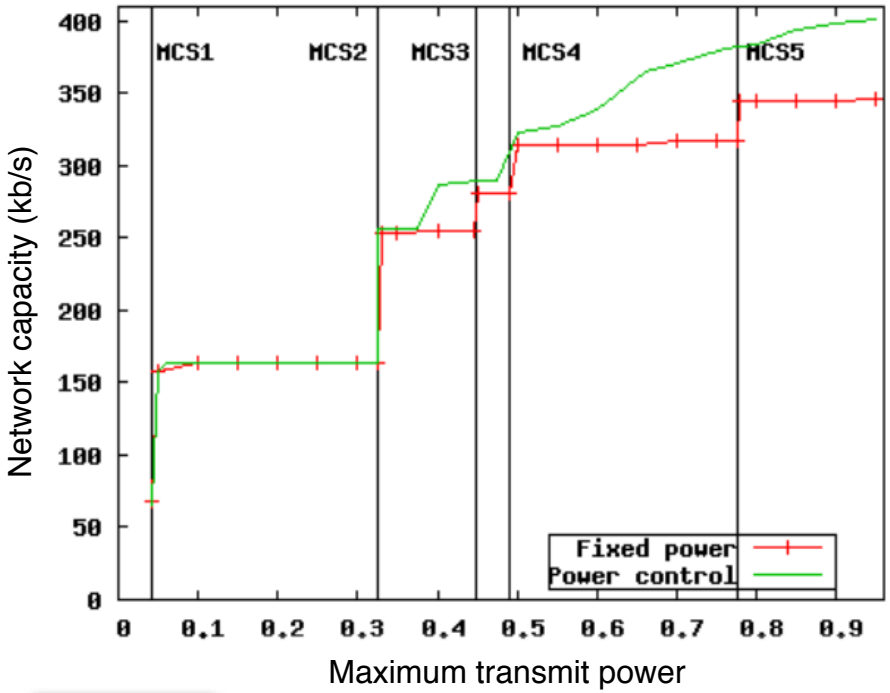
Pareto front computation

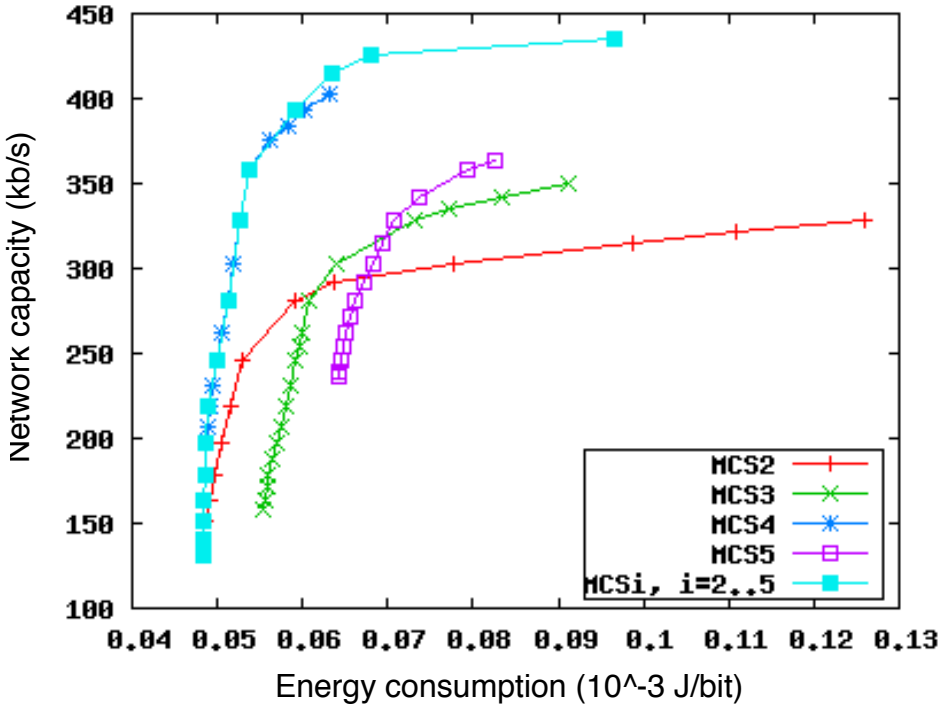
- Max capacity solution with minimum energy consumption
- Min energy solution with maximum capacity
- Iterate on energy available, get max capacity under energy budget
- Equivalently, iterate on capacity requirement

Capacity - energy Pareto front

All MCS are useful

- Reach higher capacity
- Nodes use several MCS each



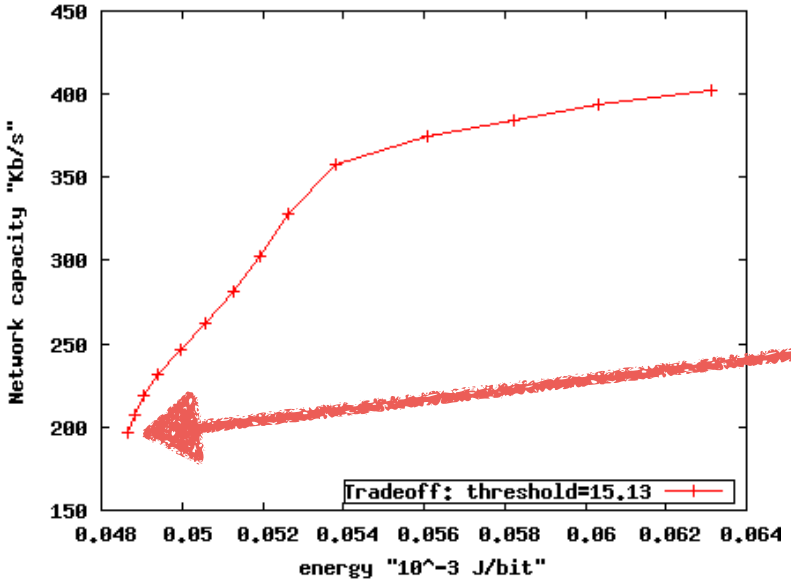


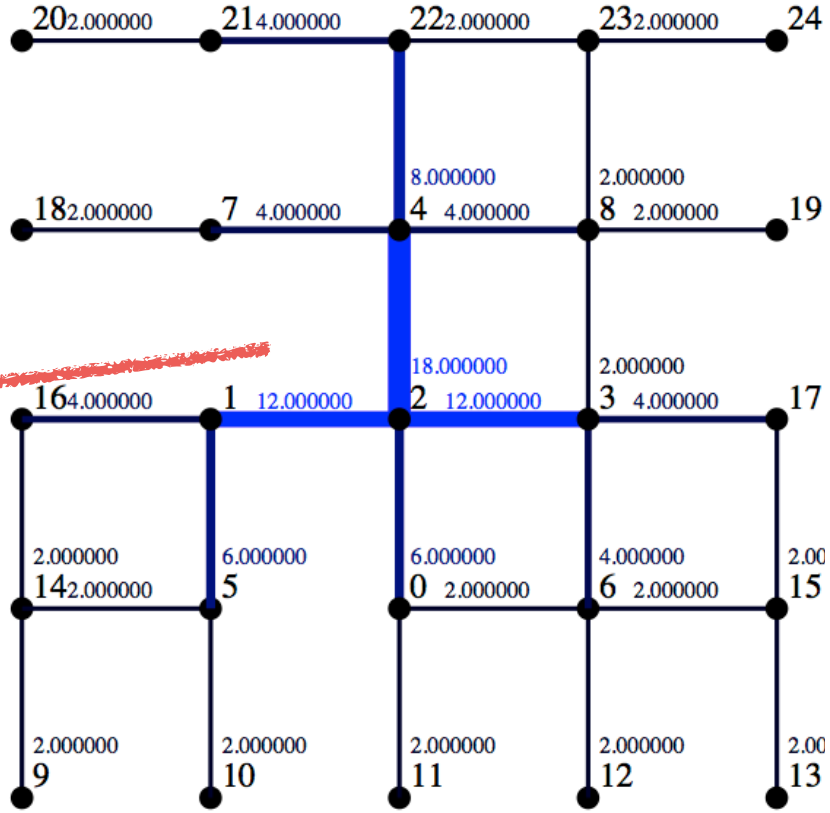
Gain of power control

- Higher max capacity
- Cost less for more

Understand optimal routing strategies

Lower energy consumption uses multi-hop

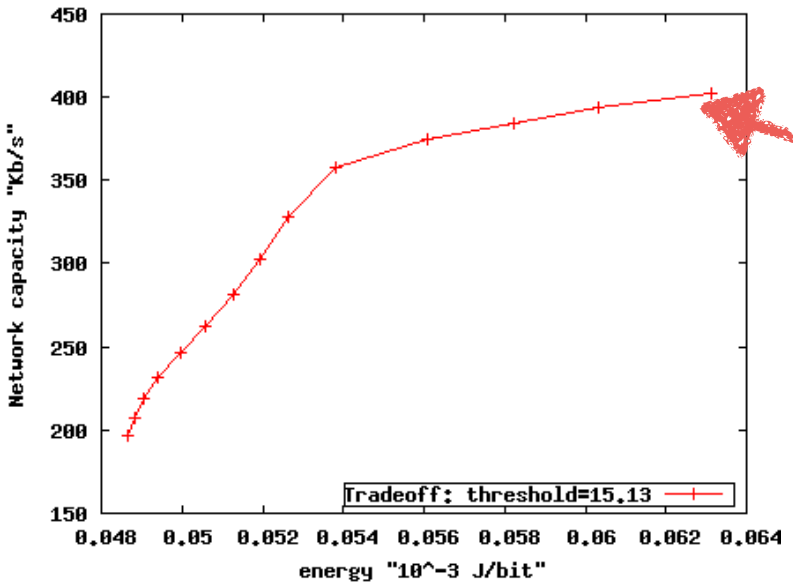


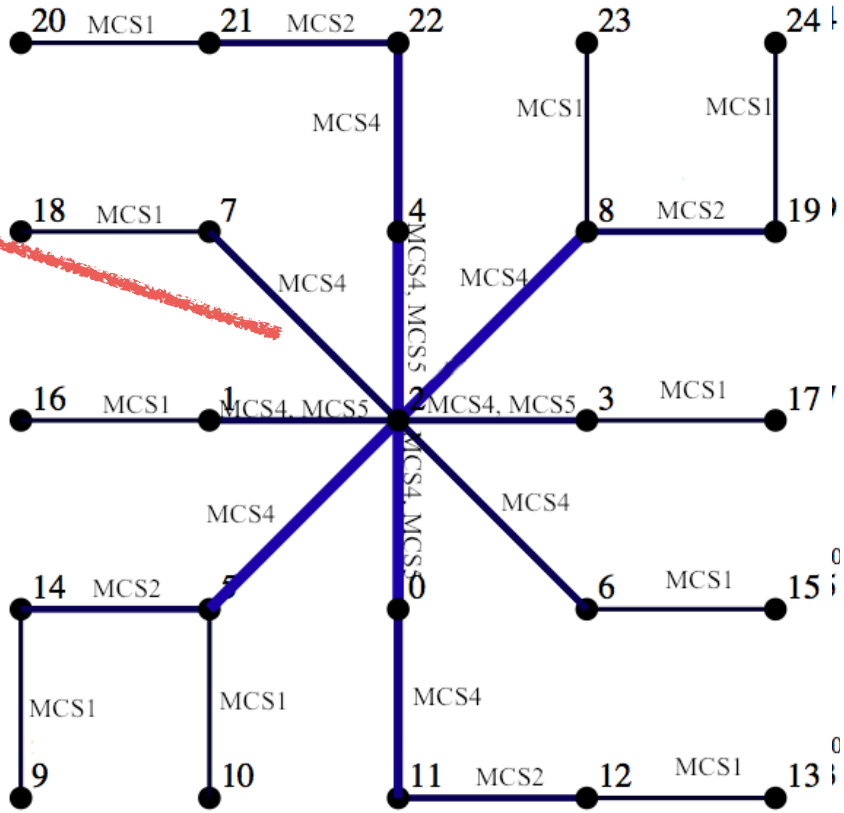


Understand optimal routing strategies

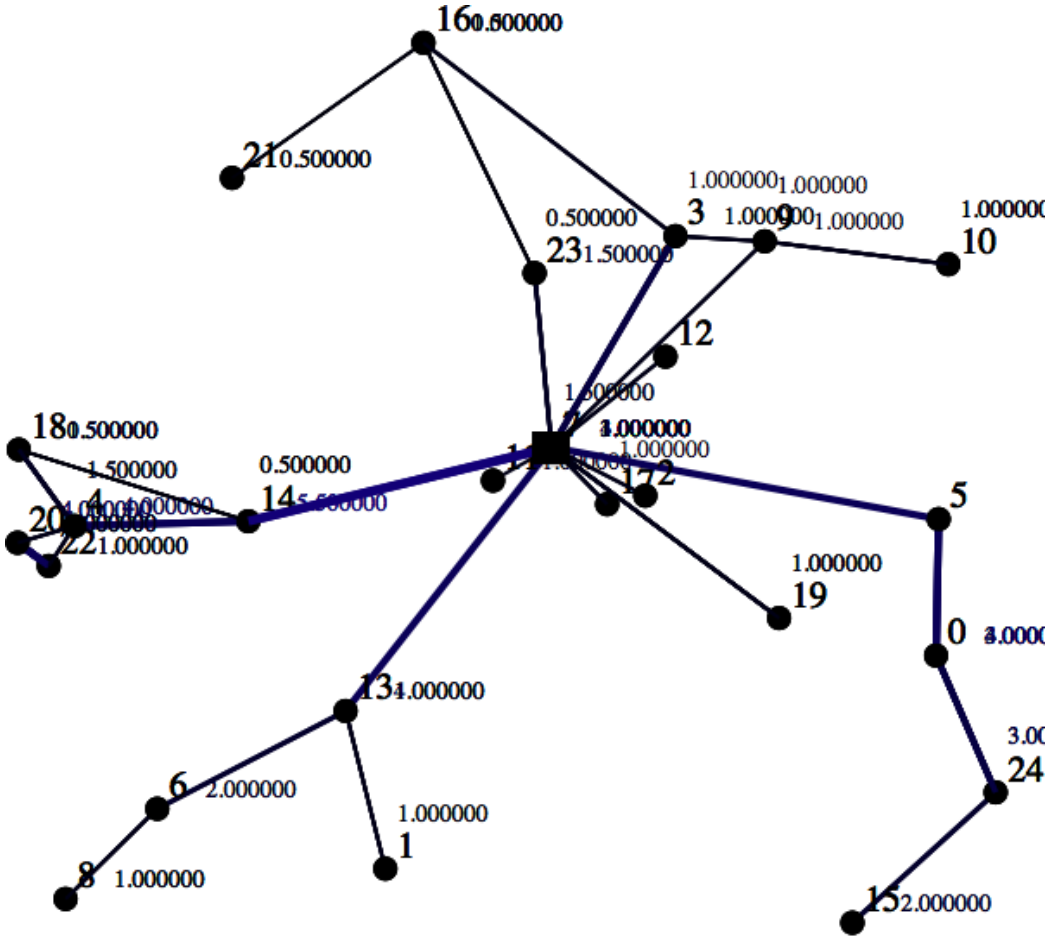
Lower energy consumption uses multi-hop

Higher capacity means short-cup in bottleneck area





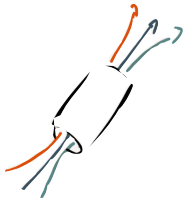
Similar conclusions on random networks



What's the point anyway?

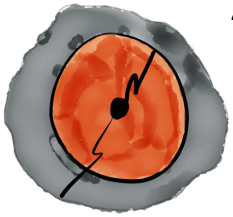
Followed by … oh wait ? The end already?

Ten years worth it ?



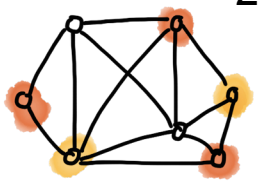
2004 : begin to take interest in radio networks

- Need to twist the problem to fit into graph theory



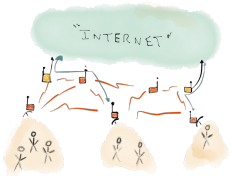
2006 : unable to compute the capacity of a network

- Combining simulation and optimization gets bounds
- Unsatisfying model gets … unsatisfying results



2009 : use of column generation to get rid of false interference models

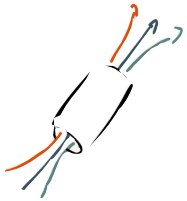
- Able to understand bottleneck effect and scaling of capacity
- Computes very fast



2013 : introduction of continuous power control and multirate in OFDMA

- Understand optimal routing strategies, power and rate allocations
- Computes capacity - « energy » tradeoffs

Ten years worth it ?



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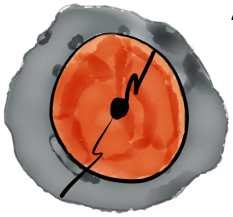

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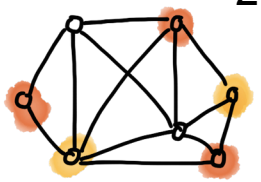

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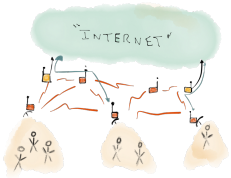


2013 : introduction of continuous power control and multirate in OFDMA

- Understand optimal routing strategies, power and rate allocations
- Computes capacity - « energy » tradeoffs

2014 : it’s not over: still many open problems to address

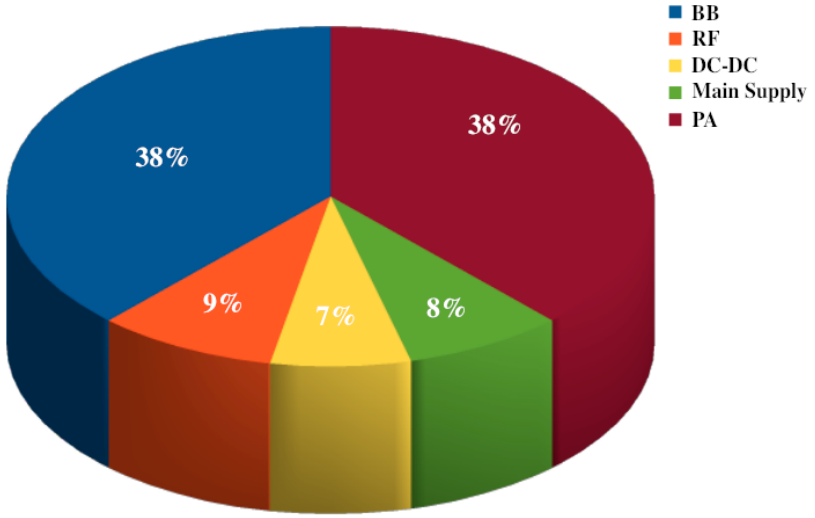
Back on energy models

« Always on » hypothesis is the core problem

- Transmit power account for 10% of global expenditure
- Only way to gain is to switch off the nodes
- Partial sleep studied by Anis Ouni
- PhD of Soukaina Cherkaoui

Leveraging power control still interesting

- Enable better capacity as seen
- Electromagnetic pollution issues
- Heterogeneous architectures



Strong hypothesis not so easy to get rid off

Dynamic traffics

- Very contradictory with a classical optimization approach
- Adapt evolving graphs and DTN for slow dynamics?
- High dynamics due to users mobility, behavior and usage

Unstable links

- Environment matters : what model of instability?
- Lossy flows taking intra/inter flows interferences?

For both case, robust optimization is a promising way

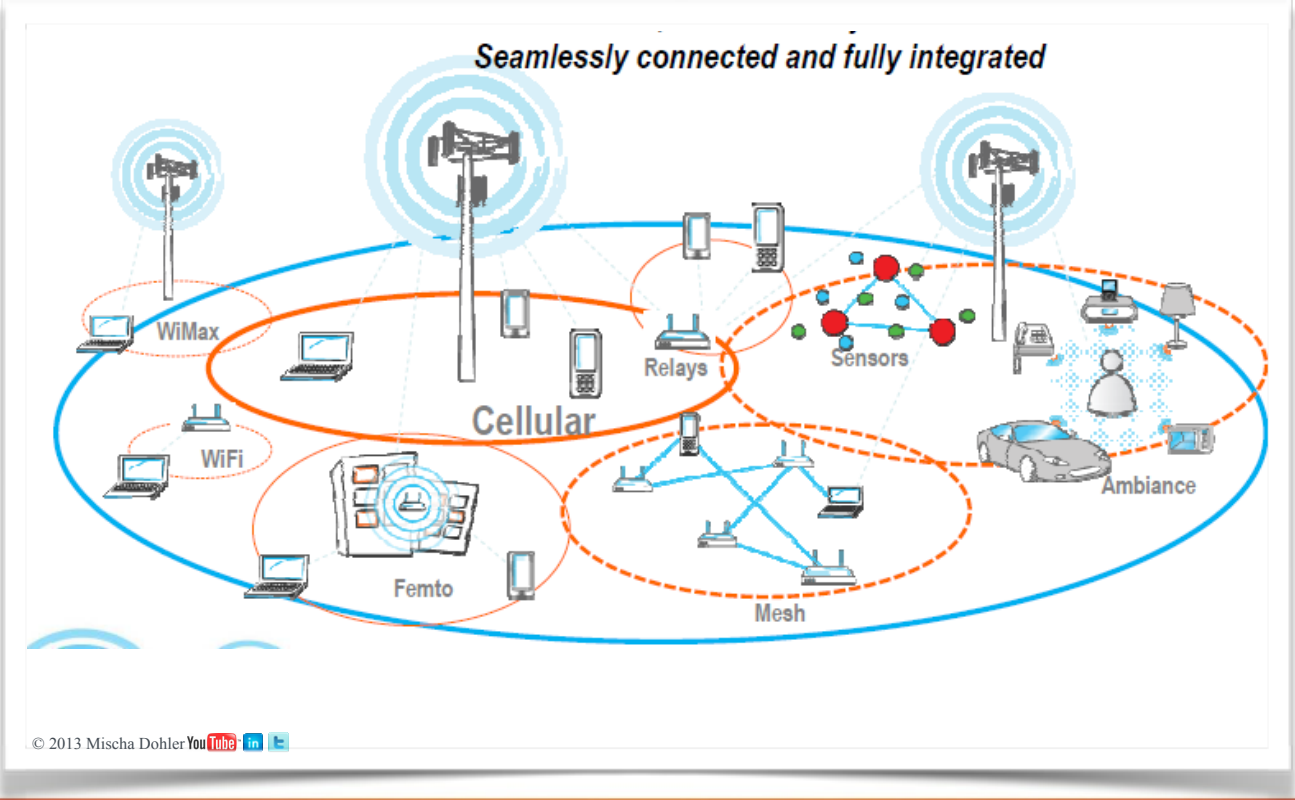
- Need for some progress on the operational research field
- Accelerating again computations would be mandatory

Grid and random networks topologies have strong properties

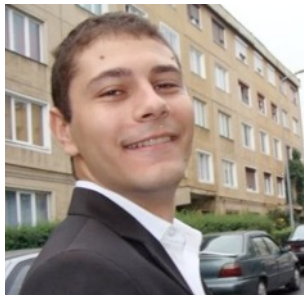
- Properties of a urban small cell deployment?

Architecture perspective: capillary networks

- Heterogeneous cellular networks
- Network offloading/coverage extension with multi-hop
- Single-hop sensor to relay followed by wireless mesh to gateways



Won’t do that alone : the UrbaNet Team



Wireless sensor networks, urban deployment

Real time, cross-layer, formal methods

Routing, resources allocation, optimization

Mobility-aware networking, network offloading

Self-*, 0-control, protocol analysis

Associated members: CNR-IEIIT Turino, U. Paris 13

Thanks to them

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