Homework 6 :Source coding

(1) Problem 1 : Lossless compression

Consider a grey image of N_p pixels. Each pixel is encoded with 8 bits. Lossless compress is evaluated.

- (1) Compute the compression factor that can be achieved when the pixels are encoded separately.
- (2) Compute the compression factor that can be achieved shen the image is encoded in a one shot.
- (3) Compare the corresponding gains.

For this study, you will not consider any spatial correlation between the pixels. Pixels are independently drawn. You will use the following conditions :

— the grey levels follow a uniform PMF.

— the grey levels follow some sampled exponential PMF.

You can use a theoretical approach or/and a simulation based approach.

(2) Problem 2 : Rate-distortion

- (1) Write the complete proof of Shannon's rate-distortion theorem, in a very comprehensive way, using the definitions and theorems presented in the course.
 - i) You can start from the lecture notes of Yuri Polianskiy, but write it from your own, with proper justification of all steps.
 - ii) Latex file is preferable.
 - iii) Illustrations are welcome.
 - iv) As well as pedagogic comments on the different calculation steps.